Engineering Mathematics-I VTU CBCS Question Paper Set 2018

CBCS Scheme

17MAT11

USN

First Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Find the n^{th} derivative of $\cos x \cos 2x$. (06 Marks)

b. Find the angle between the curves $r = a\log\theta$, $r = \dfrac{a}{\log\theta}$. (07 Marks)

c. Find the radius of curvature of the curve $r = a(1+\cos\theta)$. (07 Marks)

OR

2 a. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (06 Marks)

b. With usual notations prove that the pedal equation in the form $\dfrac{1}{p^2} = \dfrac{1}{r^2} + \dfrac{1}{r^4}\left(\dfrac{dr}{d\theta}\right)^2$. (07 Marks)

c. Find the radius of curvature of the curve $y^2 = \dfrac{a^2(a-x)}{x}$ at the point (a, 0). (07 Marks)

Module-2

3 a. Find the Taylor's series of $\log x$ in powers of $(x-1)$ upto fourth degree terms. (06 Marks)

b. If $U = \tan^{-1}\left(\dfrac{x^3+y^3}{x+y}\right)$, prove that $x\dfrac{\partial U}{\partial x} + y\dfrac{\partial U}{\partial y} = \sin 2U$ by using Euler's theorem. (07 Marks)

c. If $U = x+3y^2$, $V = 4x^2yz$, $W = 2z^2 - xy$, evaluate $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at the point (1, -1, 0). (07 Marks)

OR

4 a. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{a^x+b^x+c^x}{3}\right)^{1/x}$. (06 Marks)

b. Find the Maclaurin's expansion of $\log(\sec x)$ upto x^4 terms. (07 Marks)

c. If $z = f(x,y)$, where $x = r\cos\theta$, $y = r\sin\theta$, prove that $\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2 = \left(\dfrac{\partial z}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial\theta}\right)^2$. (07 Marks)

Module-3

5 a. A particle moves along the curve $\vec{r} = (t^3-4t)\hat{i} + (t^2+4t)\hat{j} + (8t^2-3t^3)\hat{k}$. Find the velocity and acceleration vectors at time t and their magnitudes at t = 2. (06 Marks)

b. If $\vec{f} = (x+y+1)\vec{i} + \vec{j} - (x+y)\vec{k}$, prove that $\vec{f}.\text{curl}\,\vec{f} = 0$. (07 Marks)

c. Prove that $\text{div}(\text{curl}\,\vec{A}) = 0$. (07 Marks)

OR

6 a. A particle moves along the curve $\vec{r} = 2t^2\hat{i} + (t^2 - 4t)\hat{j} + (3t - 5)\hat{k}$. Find the components o velocity and acceleration along $\hat{i} - 3\hat{j} + 2\hat{k}$ at t = 2. (06 Marks

b. If $\vec{f} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$, find div $\vec{f}$ and curl $\vec{f}$. (07 Marks

c. Prove that curl(grad ϕ) = 0. (07 Marks

Module-4

7 a. Evaluate $\int_0^{2a} \frac{x^2}{\sqrt{2ax - x^2}} dx$. (06 Marks)

b. Solve $\frac{dy}{dx} + y \tan x = y^3 \sec x$. (07 Marks)

c. Find the orthogonal trajectories of $r^n = a^n \cos n\theta$. (07 Marks)

OR

8 a. Find the reduction formula for $\int \cos^n x\, dx$ and hence evaluate $\int_0^{\pi/2} \cos^n x\, dx$. (06 Marks)

b. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. (07 Marks)

c. A body originally at 80°C cools down to 60°C in 20 minutes in the surroundings of temperature 40°C. Find the temperature of the body after 40 minutes from the original instant. (07 Marks)

Module-5

9 a. Find the rank of the matrix

$$A = \begin{pmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{pmatrix}$$

by reducing it to echelon form. (06 Marks)

b. Using the power method find the largest eigenvalue and the corresponding eigenvector of matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ taking $(1, 1, 1)^T$ as the initial eigenvector. Perform five iterations. (07 Marks)

c. Show that the transformation $y_1 = x_1 + 2x_2 + 5x_3$, $y_2 = 2x_1 + 4x_2 + 11x_3$, $y_3 = -x_2 + 2x_3$ is regular. Also, find the inverse transformation. (07 Marks)

OR

10 a. Solve the following system of equations by using Gauss-Jordan method:
$x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$ (06 Marks)

b. Diagnolize the matrix $A = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$. (07 Marks)

c. Obtain the canonical form of $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ using orthogonal transformation. (07 Marks)

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CBCS Scheme

USN | | | | | | | | | |

15MAT11

First Semester B.E. Degree Examination, June/July 2016
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the n^{th} derivative of $y = e^{-3x}\cos^3 x$. **(06 Marks)**

b. Find the angle of intersection between the curves $r = a(1 + \sin\theta)$ and $r = a(1 - \cos\theta)$. **(05 Marks)**

c. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. **(05 Marks)**

OR

2 a. If $y = \sin(\log(x^2 + 2x + 1))$, prove that $(x + 1)^2 y_{n+2} + (2n + 1)(x + 1)y_{n+1} + (n^2 + 4)y_n = 0$. **(06 Marks)**

b. Find the pedal equation for the curve $r^m \cos m\theta = a^m$. **(05 Marks)**

c. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1). **(05 Marks)**

Module-2

3 a. Expand $\sin x$ in powers of $x - \frac{\pi}{2}$ upto 4^{th} degree terms using Taylor's series. **(05 Marks)**

b. Evaluate: $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$. **(05 Marks)**

c. If $u = \tan^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$. **(06 Marks)**

OR

4 a. Expand $\log(1 + e^x)$ using Maclaurin's series upto 3^{rd} degree terms. **(06 Marks)**

b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(05 Marks)**

c. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, find $J\left(\frac{x, y, z}{r, \theta, \phi}\right)$. **(05 Marks)**

Module-3

5 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time, find the component of its velocity and acceleration in the direction of the vector $i - 3j + 2k$ at $t = 1$. **(06 Marks)**

b. Show that $\vec{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational, find ϕ such that $F = \nabla\phi$. **(05 Marks)**

c. Prove that div (curl u) = 0. **(05 Marks)**

OR

6 a. If $\vec{r} = x_i + y_j + z_k$, then prove that : i) $\nabla \times \vec{r} = 0$ ii) $\nabla^2 r^n = n(n+1)r^{n-2}$. (06 Marks)

(05 Marks)

b. Prove with usual notations Curl (grad ϕ) = 0

c. Find div $\vec{f}$ and curl $\vec{f}$ of $\vec{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. (05 Marks)

Module-4

7 a. Obtain the reduction formula of $\int \sin^m x \cos^n x \, dx$. (06 Marks)

b. Solve $(x^2 + y^3 + 6x)\,dx + y^2 x\,dy = 0$. (05 Marks)

c. Find the orthogonal trajectory of $r^n = a^n \cos n\theta$, where a is the parameter. (05 Marks)

OR

8 a. Obtain the reduction formula of $\int \cos^n x \, dx$ and hence evaluate : $\int_0^{\pi/2} \cos^n x \, dx$. (06 Marks)

b. Solve $\frac{dy}{dx} = xy^3 - xy$. (05 Marks)

c. If the temperature of the air is 30°C and the substance cools from 100°C to 70°C in 15 minutes, find when the temperature reaches at 40°C. (Use Newton's law of cooling). (05 Marks)

Module-5

9 a. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$ (06 Marks)

b. Find the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ by power method, use $[1, 0\ 0]^T$. as initial vector, take five iterations. (05 Marks)

c. Reduce the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ to the diagonal form. (05 Marks)

OR

10 a. Use Gauss – Siedel iteration method upto 3 iterations to solve with (0, 0, 0) as initial values

$10x + y + z = 12$

$x + 10y + z = 12$

$x + y + 10z = 12$. (06 Marks)

b. Show that the transformation :

$y_1 = 2x_1 + x_2 + x_3$

$y_2 = x_1 + x_2 + 2x_3$

$y_3 = x_1 - 2x_3$

is regular. Write down the inverse transformation. (05 Marks)

c. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. (05 Marks)

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CBCS Scheme

USN | | | | | | | | | | |

15MAT11

First Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Module-1

1 a. Obtain the n^{th} derivative of $\dfrac{x}{(x-1)^2(x+2)}$. **(06 Marks)**

b. Find the angle of intersection of the curves $r = a(1+\sin\theta)$ and $r = a(1-\sin\theta)$. **(05 Marks)**

c. Find the radius of curvature at the point $\left(\dfrac{3a}{2},\dfrac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. **(05 Marks)**

OR

2 a. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then prove that $(x^2-1)\,y_{n+2} + (2n+1)\,xy_{n+1} + (n^2-m^2)y_n = 0$. **(06 Marks)**

b. Obtain the pedal equation of the curve $r^n = a^n \cos n\theta$. **(05 Marks)**

c. Find the derivative of arc length of $x = a\left(\cos t + \log\tan\left(\frac{t}{2}\right)\right)$ and $y = a\sin t$. **(05 Marks)**

Module-2

3 a. Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e(1.1)$, correct to four decimal places. **(06 Marks)**

b. If $z = \sin(ax+y) + \cos(ax-y)$, prove that $\dfrac{\partial^2 z}{\partial x^2} = a^2\dfrac{\partial^2 z}{\partial y^2}$. **(05 Marks)**

c. If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$, then find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **(05 Marks)**

OR

4 a. If $u(x+y) = x^2+y^2$, then prove that $\left(\dfrac{\partial u}{\partial x}-\dfrac{\partial u}{\partial y}\right)^2 = 4\left(1-\dfrac{\partial u}{\partial x}-\dfrac{\partial u}{\partial y}\right)$. **(06 Marks)**

b. Evaluate $\displaystyle\operatorname*{Lt}_{x\to 0}\left(\frac{a^x+b^x+c^x+d^x}{4}\right)^{1/x}$. **(05 Marks)**

c. If $u = f\left(\dfrac{x}{y},\dfrac{y}{z},\dfrac{z}{x}\right)$, then prove that $x\,u_x + y\,u_y + z\,u_z = 0$. **(05 Marks)**

Module-3

5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. **(06 Marks)**

b. If $\vec{f} = (x+y+az)\hat{i} + (bx+2y-z)\hat{j} + (x+cy+2z)\hat{k}$, find a, b, c such that $\vec{f}$ is irrotational. **(05 Marks)**

c. Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at the point P(2, -1, 2). **(05 Marks)**

OR

6 a. Find the directional derivative of $xy^3 + yz^3$ at (2, -1, 1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. **(06 Marks)**

b. If $\vec{u} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ and $\vec{v} = yz\hat{i} + zx\hat{j} + xy\hat{k}$, show that $\vec{u} \times \vec{v}$ is a solenoidal vector. **(05 Marks)**

c. For any scalar field ϕ and any vector field $\vec{f}$, prove that curl $(\phi \vec{f}) = \phi$ curl $\vec{f}$ + (grad ϕ) $\times \vec{f}$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \cos^n x \; dx$, where n is a positive integer, hence evaluate $\int_0^{\pi/2} \cos^n x dx$. **(06 Marks)**

b. Solve : $(x^2 + y^2 + x)\, dx + xydy = 0$. **(05 Marks)**

c. Find the orthogonal trajectories of the family of circles $r = 2a\cos\theta$, where 'a' is a parameter. **(05 Marks)**

OR

8 a. Evaluate $\int_0^\infty \frac{x^6}{(1+x^2)^{9/2}} dx$. **(06 Marks)**

b. Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$. **(05 Marks)**

c. Water at temperature 10^0C takes 5 minutes to warm upto 20^0C in a room temperature 40^0C. Find the temperature after 20 minutes. **(05 Marks)**

Module-5

9 a. Solve the following system of equations by Gauss Elimination Method. **(06 Marks)**
$x + 2y + z = 3$, $2x + 3y + 2z = 5$, $3x - 5y + 5z = 2$.

b. Find the dominant eigen value and the corresponding eigen vector by power method $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, perform 5 iterations, taking initial eigen vector as $[1\ 1\ 1]^T$. **(05 Marks)**

c. Show that the transformation $y_1 = 2x + y + z$, $y_2 = x + y + 2z$, $y_3 = x - 2z$ is regular. Write down the inverse transformation. **(05 Marks)**

OR

10 a. Solve the following system of equations by Gauss – Seidel method. **(06 Marks)**
$10x + 2y + z = 9$, $x + 10y - z = -22$, $-2x + 3y + 10z = 22$.

b. Reduce the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ to the diagonal form. **(05 Marks)**

c. Reduce $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form. **(05 Marks)**

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CBCS Scheme

USN | | | | | | | | | | | **15MAT11**

First Semester B.E. Degree Examination, Dec.2015/Jan.2016
Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Find the n^{th} derivative of $\dfrac{x^2}{2x^2+7x+6}$. **(06 Marks)**

b. Find the angle between the curves $r^2 \sin 2\theta = 4$ and $r^2 = 16 \sin 2\theta$. **(05 Marks)**

c. Find the radius of curvature of the curve represented by $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. **(05 Marks)**

OR

2 a. If $y = (x + \sqrt{x^2 - 1})^m$ then prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$. **(06 Marks)**

b. Find the pedal equation of $r^n = a(1 + \cos n\theta)$. **(05 Marks)**

c. Find the radius of curvature of the curve $r^n = a^n \sin n\theta$. **(05 Marks)**

Module-2

3 a. Expand $\sin x$ in powers of $(x - \frac{\pi}{2})$ upto fourth degree term. **(06 Marks)**

b. Evaluate $\lim\limits_{x\to 0} \dfrac{xe^x - \log(1+x)}{x^2}$. **(05 Marks)**

c. If $u = x + y + z$, $uv = y + z$, $uvw = z$ then find $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$. **(05 Marks)**

OR

4 a. Find the Maclaurin's series expansion of $\sec x$ upto x^4 term. **(06 Marks)**

b. If $V(x,y) = (1-2xy + y^2)^{-1/2}$ and $x\dfrac{\partial v}{\partial x} - y\dfrac{\partial v}{\partial y} = y^2 V^K$, then find K. **(05 Marks)**

c. If $u = \sin^{-1}\left\{\dfrac{x+2y+3z}{x^8+y^8+z^8}\right\}$ then find $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z}$. **(05 Marks)**

Module-3

5 a. A particle moves along the curve whose parametric equations are $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the component of its velocity at t = 1 in the direction of I + J + 3K. Find also the component of its acceleration at t = 1 along the normal to I + J + 3K. **(06 Marks)**

b. Verify whether $\vec{A} = (2x + yz)\,I + (4y + zx)\,J - (6z - xy)K$ is irrotational or not. And find the scalar potential of $\vec{A}$. **(05 Marks)**

c. If $\vec{A}$ is a vector point function and ϕ is a scalar point function then prove that $\text{div}(\phi\vec{A}) = \phi\,\text{div}\,\vec{A} + (\text{grad}\,\phi)\cdot\vec{A}$. **(05 Marks)**

OR

6 a. If $\vec{f} = x^2 I + y^2 J + z^2 K$ and $\vec{g} = yzI + zxJ + xyK$, then verify whether $\vec{f} \times \vec{g}$ is solenoidal or not. **(06 Marks)**

b. Find the directional derivative of $\phi = x^2 + y^2 + 2z^2$ at P(1, 2, 3) in the direction of line $\overrightarrow{PQ} = 4i - 2j + k$. **(05 Marks)**

c. Prove that curl (grad ϕ) = $\vec{O}$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^n x\, dx$. Hence evaluate $\int_0^{\pi/2} \sin^n x\, dx$. **(06 Marks)**

b. Solve $(4xy + 3y^2 - x)\, dx + x(x+2y)dy = 0$. **(05 Marks)**

c. Find the Orthogonal trajectories of the family $r^n = a^n \sin n\theta$, where a is the parameter. **(05 Marks)**

OR

8 a. Evaluate $\int_0^{\infty} \frac{x^6 dx}{(4+x^2)^{15/2}}$. **(06 Marks)**

b. Solve $x\frac{dy}{dx} + y = x^3 y^6$. **(05 Marks)**

c. A body is heated to 110^0C and placed in air at 10^0C. After one hour its temperature become 60^0C. How much additional time is required for it to cool to 30^0C? **(05 Marks)**

Module-5

9 a. Solve the following system of equations by Gauss – Jordan method :
$x + y + z = 8$; $-x - y + 2z = -4$; $3x + 5y - 7z = 14$. **(06 Marks)**

b. Verify the transformation $y_1 = 19x_1 - 9x_2 + 2x_3$; $y_2 = -4x_1 + 2x_2 - x_3$; $y_3 = -2x_1 + x_2$ is regular or not and find the inverse transformation if possible. **(05 Marks)**

c. Reduce the matrix to the diagonal form
$$A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}.$$ **(05 Marks)**

OR

10 a. Solve the following system by Gauss – Seidal method : **(06 Marks)**
$20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$. Perform three iterations.

b. Determine the largest eigen value and the corresponding eigen vector of
$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$ using Power method. **(05 Marks)**
Take $(1, 0, 0)^T$ as the initial eigen vector and perform four iterations.

c. Reduce the quadratic form :
$8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form. **(05 Marks)**

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15MAT11

First Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Module-1

1 a. If $y = e^{-3x}\cos^3 x$, find y_n. **(06 Marks)**

b. Find the angle between the curves $r = \dfrac{a}{1+\cos\theta}$ and $r = \dfrac{b}{1-\cos\theta}$. **(05 Marks)**

c. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1). **(05 Marks)**

OR

2 a. If $x = \tan(\log y)$, find the value of $(1+x^2)y_{n+1} + (2nx-1)y_n + (n)(n-1)y_{n-1}$. **(06 Marks)**

b. Find the Pedal equation of $\dfrac{2a}{r} = 1 + \cos\theta$. **(05 Marks)**

c. Find the radius of curvature of the curve $r^n = a^n \cos n\theta$. **(05 Marks)**

Module-2

3 a. Explain log(cos x) about the point $x = \dfrac{\pi}{3}$ upto 3rd degree terms using Taylor's series. **(06 Marks)**

b. Evaluate $\displaystyle\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$. **(05 Marks)**

c. State Euler's theorem and use it to find $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ when $u = \tan^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$. **(05 Marks)**

OR

4 a. Expand $\dfrac{e^x}{1+e^x}$ using Maclaurin's series upto and including 3rd degree terms. **(06 Marks)**

b. Find $\dfrac{du}{dt}$ when $u = x^3y^2 + x^2y^3$ with $x = at^2$, $y = 2at$. Use Partial derivatives. **(05 Marks)**

c. If $u = \dfrac{x_2x_3}{x_1}$, $v = \dfrac{x_1x_3}{x_2}$, $w = \dfrac{x_1x_2}{x_3}$, find the value of Jacobian $J\left(\dfrac{u, v, w}{x_1, x_2, x_3}\right)$. **(05 Marks)**

Module-3

5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time find the components of velocity and acceleration at time t = 1 in the direction of $i - 3j + 2k$. **(06 Marks)**

b. Find the divergence and curl of the vector $\vec{V} = (xyz)i + (3x^2y)j + (xz^2 - y^2z)K$ at the point (2, -1, 1). **(05 Marks)**

c. A vector field is given by $\vec{A} = (x^2 + xy^2)\,i + (y^2 + x^2y)j$, show that the field is irrotational and find the scalar potential. **(05 Marks)**

Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

6 a. Find grad ϕ when $\phi = 3x^2y - y^3z^2$ at the point (1, -2, -1). **(06 Marks)**

b. Find a for which f = (x + 3y)i + (y - 2z)j + (x + az)k is solenoidal. **(05 Marks)**

c. Prove that Div(curl $\vec{V}$) = 0. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula of $\int \sin^m x \cos^n x \, dx$. **(06 Marks)**

b. Evaluate $\int_0^{2a} x\sqrt{2ax - x^2}\, dx$. **(05 Marks)**

c. Solve $(2x \log x - xy)\, dy + 2y\, dx = 0$. **(05 Marks)**

OR

8 a. Obtain the reduction formula of $\int \cos^n x \, dx$. **(06 Marks)**

b. Obtain the Orthogonal trajectory of the family of curves $r^n \cos n\theta = a^n$. Hence solve it. **(05 Marks)**

c. A body originally at 80^0C cools down at 60^0C in 20 minutes, the temperature of the air being 40^0C. What will be the temperature of the body after 40 minutes from the original? **(05 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$

(06 Marks)

b. Solve by Gauss – Jordan method the system of linear equations
$2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$. **(05 Marks)**

c. Find the largest eigen value and the corresponding Eigen vector by power method given that $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. (Use $[1\ 0\ 0]^T$ as the initial vector). (Apply 4 iterations). **(05 Marks)**

OR

10 a. Use Gauss – Seidel method to solve the equations
$20x + y - 2x = 17$
$3x + 20y - z = 18$
$2x - 3y + 20z = 25$. Carry out 2 iterations with $x_0 = y_0 = z_0 = 0$. **(06 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form. **(05 Marks)**

c. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. **(05 Marks)**

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CBCS Scheme

USN

15MAT11

First Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Module-1

1 a. Find the n^{th} derivative of $y = e^{-x}\sin x\cos 2x$. **(06 Marks)**

b. Show that the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ cut each other orthogonally. **(05 Marks)**

c. Find the radius of curvature of the curve $x^2y = a(x^2+y^2)$ at the point (-2a, 2a). **(05 Marks)**

OR

2 a. If $y = \sin(m\sin^{-1}x)$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$ **(06 Marks)**

b. Find the pedal equation of $r = 2(1+\cos\theta)$. **(05 Marks)**

c. Find the radius of curvature of $r^n = a^n\sin n\theta$. **(05 Marks)**

Module-2

3 a. Expand $\tan^{-1}x$ in powers of $(x-1)$ upto the fourth degree term. **(06 Marks)**

b. Evaluate $\lim_{x\to 0}\left[\frac{1}{x} - \frac{\log(1+x)}{x^2}\right]$ **(05 Marks)**

c. If $z = f(x+ct) + g(x-ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = C^2 . \frac{\partial^2 z}{\partial x^2}$. **(05 Marks)**

OR

4 a. Obtain the Maclaurin's series expansion of $e^{\sin x}$ upto the form containing x^4. **(06 Marks)**

b. If $z = \log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$. **(05 Marks)**

c. If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$. **(05 Marks)**

Module-3

5 a. A particle moves along the curve whose parametric equations are $x = t^3+1$, $y = t^2$ and $z = 2t+5$. Find the components of its velocity and acceleration at time t = 1 in the direction of $i + j + 3k$. **(06 Marks)**

b. If $\phi = 2x^3y^2z^4$, find $\text{Div}(\text{Grad}\,\phi)$. **(05 Marks)**

c. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ, such that $\vec{F} = \nabla\phi$. **(05 Marks)**

OR

6 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at P(1, –2, –1) in the direction of $2i - j - 2k$. **(06 Marks)**

b. If $\vec{F} = (x+y+1)i + j - (x+y)k$. Show that $\vec{F}.\text{curl}\,\vec{F} = 0$. **(05 Marks)**

c. If $\vec{F} = \nabla(xy^3z^2)$, find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$ at the point (1, –1, 1). **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \cos^n x\,dx$. **(06 Marks)**

b. Solve $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$. **(05 Marks)**

c. Find the orthogonal trajectories of the family of curves $y^2 = Cx^3$. **(05 Marks)**

OR

8 a. Evaluate $\int_0^1 x^{3/2}(1-x)^{3/2}dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} - \frac{2}{x}y = \frac{y^2}{x^3}$. **(05 Marks)**

c. A body is heated to 110°C and placed in air at 10°C. After one hour its temperature becomes 60°C. How much additional time is required for it to cool to 30°C? **(05 Marks)**

Module-5

9 a. Find the rank of the matrix $A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$. **(06 Marks)**

b. Solve the following system of equations by Gauss Jordan method:
$x + 2y + z = 3$, $2x + 3y + 3z = 10$, $3x - y + 2z = 13$ **(05 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. **(05 Marks)**

OR

10 a. Solve the following system of equations by Gauss-Seidal method:
$20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$. Perform three iterations. **(06 Marks)**

b. Show that the transformation, $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular and find the inverse transformation. **(05 Marks)**

c. Reduce the quadratic form,
$3x^2 + 3y^2 + 3z^2 + 2xy - 2yz + 2zx$ into the canonical form. **(05 Marks)**

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