Assignment Problems

MODULE - I

DIFFERENTIAL CALCULUS- I

1. Find the n[th] derivative of the following functions**:**

(i). $\frac{6x}{(x-1)(x^2-4)}$ (ii). $\frac{x}{(x+2)(x^2-2x+1)}$ (iii). $\frac{x^2+4x+1}{x^3+2x^2-x-2}$ (iv). $\frac{x}{4x^2-x-3}$

(v). $\frac{x^3}{x^2-3x+2}$ (vi). $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ (vii). $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

2. Leibnitz Theorem Problems:

i. show that $\frac{d^n}{dx^n}\left[\frac{\log x}{x}\right]=\frac{(-1)^n n!}{x^{n+1}}\left[\log x-1-\frac{1}{2}-\frac{1}{3}-.........\frac{1}{n}\right]$

Hint: Take $v=\log x;\ u=\frac{1}{x}$

ii. If $y=(x^2-1)^n$, Show that y_n satisfies the equation

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$

Hint : It is required to show that

$$\left(1-x^2\right)y_{n+2}-2xy_{n+1}+n\left(n+1\right)y_n=0$$

iii. If $y=a\cos(\log x)+b\sin(\log x)$,

$$x^2y_{n+2}+\left(2n+1\right)xy_{n+1}+\left(n^2+1\right)y_n=0$$

iv. If $y=e^{m\sin^{-1}x}$, Prove That

$$\left(1-x^2\right)y_{n+2}-\left(2n+1\right)xy_{n+1}-\left(n^2+m^2\right)y_n=0$$

v. If $\cos^{-1}\left(\frac{y}{b}\right)=\log\left(\frac{x}{n}\right)^n$, Show that

$$x^2y_{n+2}+\left(2n+1\right)xy_{n+1}+2n^2y_n=0$$

vi. $y = \sin(m \sin^{-1} x)$, Prove That

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$$

3. Find the acute angle between the curves

(i) $r^n = a^n(\cos n\theta + \sin n\theta)$ and $r^n = a^n \sin n\theta$

(ii) $r^n \cos n\theta = a^n$ and $r^n \sin n\theta = b^n$

(iii) $r = a\theta$ and $r = a/\theta$

(iv) $r = a\cos\theta$ and $r = a/2$

(v) $r^m = a^m \cos m\theta$ and $r^m = b^m \sin m\theta$

4. Find the pedal equations of the following polar curves

(i) $r^n \cos n\theta = a^n$

(ii) $r = a\theta$

(iii) $r = a\cos\theta$

(iv) $r^m = b^m \sin m\theta$

5. Find the radius of curvature for the following curves

(i) $x = a(t - \sin t), \quad y = a(1-\cos t)$

(ii) $x = a\cos\theta, \quad y = b\sin\theta \quad at (a/\sqrt{2}, b/\sqrt{2})$

6. Find the radius of curvature for the following curves:

(i) $x^3 + y^3 = 3axy \; at \left(\frac{3a}{2}, \frac{3a}{2}\right)$

(ii) $y^2 = \frac{a^2(a-x)}{x} \; at \; (a,0)$

(iii) $a^2 y = x^3 - a^3$ *at the point where it crosses x – axis*

(iv) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ *at the ends of the major axis.*

(v) $a^2 y = x^3 - a^3$ *at the point* $(a,0)$.

(vi) $y^2 = 2x(3-x^2)$ *at the point where the tangents are parallel to x – axis.*

(vii) $x^2 y = a(x^2+y^2)$ at $(-2a, 2a)$.

(viii) *For the curve* $y = \frac{ax}{a+x}$ *show that* $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$

7. Find the rdius of curvature of the curve $\theta = \frac{\sqrt{r^2 - a^2}}{a} - \cos^{-1}\left(\frac{a}{r}\right)$ at any point on it.

8. If ρ_1 *and* ρ_2 be the radii of curvatures at the extremities of the polar chord of the cardiode $r = a(1+\cos\theta)$, show that $\rho_1{}^2 + \rho_2{}^2 = 16a^2/9$

9. Obtain the pedal equation of the curve $r = a(1-\cos\theta)$ and hence show that $\rho = (2/3)\sqrt{2ar}$

10. Using the pedal formula for ρ, prove that $\rho = r^3/a^2$ for the curve $r^2 = a^2\sec 2\theta$

MODULE - II

DIFFERENTIAL CALCULUS- II

1 . Expand tanx about the point $x = \pi / 4$ upto the third degree terms and hence find $\tan 46^0$

2. Obtain the Maclaurin's expansion of the following functions

$i)\log(1+\tan x)$ *upto the third degree terms*

$ii)\ e^x \sin x$ *upto the fifth degree terms*

$iii)\sqrt{1+\sin 2x}$ *upto the fourth degree terms*

$iv)\ e^{x\cos x}$ *upto the fourth degree terms*

3 Evaluate the following limits.

$(i)\ \lim_{x\to 0}\frac{e^x - e^{-x} - 2\log(1+x)}{x\sin x}$ $(ii)\ \lim_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x}$

$(iii)\ \lim_{x\to 0}\frac{1+\sin x - \cos x + \log(1-x)}{x\tan^2 x}$ $(iv)\ \lim_{x\to \frac{\pi}{2}}\frac{\log\sin x}{(x-\frac{\pi}{2})^2}$

$(v)\ \lim_{x\to 0}\frac{\cosh x + \log(1-x) - 1 + x}{x^2}$ $(vi)\ \lim_{x\to \frac{\pi}{2}}\frac{\sin x \sin^{-1} x}{x^2}$

$(vii)\ \lim_{x\to 0}\frac{e^{2x} - (1+x)^2}{x\log(1+x)}$

4 Evaluate the following limits.

$(i)\ \lim_{x\to 0}\frac{\log\tan x}{\log x}$ $(ii)\ \lim_{x\to 0}\frac{\log\sin x}{\cot x}$

$(iii)\ \lim_{x\to 0}\frac{\cot 2x}{\cot 3x}$ $(iv)\ \lim_{x\to \frac{\pi}{2}}\frac{\sec x}{\tan 3x}$ $(v)\lim_{x\to 0}\log_{\sin x}\sin 2x$

5 Evaluate the following limits.

$(i)\ \lim_{x\to a}\left(2-\frac{x}{a}\right)\cot(x-a)$ $(ii)\ \lim_{x\to 0}\left(\cos ecx-\cot x\right)$

$(iii)\ \lim_{x\to\frac{\pi}{2}}\left[x\tan x-\frac{\pi}{2}\sec x\right]$ $(iv)\ \lim_{x\to 0}\left(\cot^2 x-\frac{1}{x^2}\right)$

$(v)\ \lim_{x\to 0}\left[\frac{1}{x^2}-\frac{1}{x\tan x}\right]$ $(vi)\ \lim_{x\to\frac{\pi}{2}}\left[2x\tan x-\pi\sec x\right]$

6 Evaluate the following limits.

$(i)\ \lim_{x\to 0}(\cos ax)^{\frac{b}{x^2}}$ $(ii)\ \lim_{x\to 0}\left(\frac{1+\cos x}{2}\right)^{\frac{1}{x^2}}$

$(iii)\ \lim_{x\to 1}(1-x^2)^{\frac{1}{\log(1-x)}}$ $(iv)\ \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$

$(v)\ \lim_{x\to 0}(\sin x)^{\tan x}$ $(iv)\ \lim_{x\to 0}\left(1+\sin x\right)^{\cot x}$

$(vii)\ \lim_{x\to 0}(\cos x)^{\cos ec^2 x}$ $(viii)\ \lim_{x\to\frac{\pi}{4}}\left(\tan x\right)^{\tan 2x}$

$(ix)\ \lim_{x\to\infty}(\frac{ax+1}{ax-1})^x$ $(x)\ \lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$

7 If u= $e^{ax-by}\sin(ax+by)$ show that $b\frac{\partial u}{\partial x}-a\frac{\partial u}{\partial y}=2abu$

8 If $u=\tan^{-1}\left(\frac{y}{x}\right)$ verify that

$\frac{\partial^2 u}{\partial y\partial x}=\frac{\partial^2 u}{\partial x\partial y}$ and $\frac{\partial^3 u}{\partial y^2\partial x}=\frac{\partial^3 u}{\partial x\partial y^2}=\frac{\partial^3 u}{\partial y\partial x\partial y}$

9. If u= $f\left(x+ct\right)+g(x-ct)$ $provethat$ $u_{tt}=c^2u_{xx}$

10. If $u=\log(\tan x+\tan y+\tan z)$, $show\ that$ $\sin 2xu_x+\sin 2yu_y+\sin 2zu_z=2$

11. If $r^2=x^2+y^2+z^2$, $show\ that$ $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)e^r=\left(1+\frac{2}{r}\right)e^r$

12. $Find \frac{dy}{dx}$ in $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

$if\ \ z = f(x,y)\ where\ x = r\cos\theta\ and\ y = r\sin\theta$

13. $show\ that\ \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$

$if\ \ u = e^x \sin y,\ v = e^x \cos y\ \ and\ \ w = f(u,v)\ prove\ that$

14. $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 w}{\partial u^2} + \frac{\partial^2 w}{\partial v^2}\right)$

15 . $x = r\cos\theta,\ y = r\sin\theta$ show that $J\left(\frac{x,y}{r,\theta}\right) \times J^1\left(\frac{r,\theta}{x,y}\right) = 1$

16 .If $x = e^u \sec v,\ y = e^u \tan v,$ show that $\frac{\partial(x,y)}{\partial(u,v)} \times \frac{\partial(u,v)}{\partial(x,y)} = 1$

17 .If $u = xyz, v = xy + yz + zx, w = x + y + z$ show that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = (x-y)(y-z)(z-x)$$

12.If $x + y + z = u,\ y + z = v,\ z = uvw,$ find the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

13.If $u + v = e^x \cos y, u - v = e^x \sin y$ find $J\left(\frac{u,v}{x,y}\right)$

14. Find Taylor's series expansion of $e^x \log(1+y)$ about the origin.

MODULE - III

VECTOR CALCULUS

1. If $\phi = x^2 + y^2 + z^2$ and $\vec{F} = x^2\,\vec{i} + y^2\,\vec{j} + z^2\,\vec{k}$, then find $grad\phi, div\vec{F}, curl\vec{F}$. Prove that $divCurlF = \nabla.\nabla\times F = 0$.
2. Find the directional derivative of $\phi = xy + yz + zx$ at (1,2,3) along 3i+4j+5k
3. Find the directional derivative of $\phi = e^{2x}\cos yz$ at the origin in the direction of the tangent to the curve x=asint, y=accost, z=at at t=π/4
4. Show that the surfaces $4x^2 + z^3 = 4$ *and* $5x^2 - 2yz - 9x = 0$ intersect each other orthogonally.
5. If $\vec{A} = x^2yi - 2xzj + 2yzk$, find $curl(curl\,\vec{A})$ and verify that $curl(curl\,\vec{A}) = grad(div\,\vec{A}) - \nabla^2\vec{A}$
6. If $\vec{F} = 2xy^3z^4\,\vec{i} + 3x^2y^2z^4\,\vec{j} + 4x^2y^3z^3\,\vec{k}$, find $i)\ \left(\nabla.\vec{F}\right)\ \ ii)\ \left(\nabla\times\vec{F}\right)$.
7. Find the constants a,b,c so that the vector function $\vec{F} = \left(x + 2y + az\right)\vec{i} + \left(bx - 3y - z\right)\vec{j} + \left(4x + y + 2z\right)\vec{k}$ is irrotational.
8. Find the constant 'a' so that the vector function $\vec{A} = y(ax^2 + z)i + x(y^2 - z^2)j + 2xy(z - xy)k$ is solenoidal..
9. Prove that $graddivF = curlcurlF + \nabla^2F$.

MODULE - IV

INTEGRAL CALCULUS

1. Evaluate $\int_0^{\pi} \frac{\log(1+\sin a\cos x)}{\cos x}dx$ by differentiating under the integral sign.
2. Evaluate $\int_0^1 \frac{x^{\alpha}-1}{\log x}dx \;\; (\alpha \ge 0)$ by differentiating under the integral sign. Hence find $\int_0^1 \frac{x^3-1}{\log x}dx$
3. Using Leibnitz rule under differentiation under integral sign,evaluate $\int_0^{\pi} \frac{\log(1+2\cos x)}{\cos x}dx, \alpha \ge 0$
4. Obtain the reduction formula of the integral $\int \cos^n xdx$.
5. Derive the reduction formula for $\int_0^{\pi/2} \sin^n xdx$.
6. Evaluate $\int_0^{\pi/2} \sin^7 \theta \cos^6 \theta d\theta$.
7. Evaluate $\int_0^{\infty} \frac{x^4}{(1+x)^4}dx$
8. Evaluate $\int_0^{\infty} x^5 \sin^{-1} xdx$
9. Evaluate $\int_0^{\infty} x\sin^5 x\cos^6 xdx$
10. Find the volume of the solid obtained by revolving the Astraid $x^{2/3}+y^{2/3}=a^{2/3}$ about x-axis.
11. Find the surface generated by revolving the cycloid $x=a(\theta-\sin\theta), y=a(1-\cos\theta)$ about its base,(consider on arc in the 1st quadrant) .
12. Compute the perimeter of the cardiod $r=a(1+\cos\theta)$.
13. Solve $\frac{dy}{dx}=\frac{y}{x-\sqrt{xy}}$.
14. Solve $\frac{dy}{dx}+xsn2y=x^3\cos^2 y$.

15. Solve $\left(1+x^2\right)dx+\left(x-e^{-\tan^{-1}y}\right)dy=0$.

16. Solve $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xye^{xy^2}-3y^2\right)dy=0$.

17. Solve $\left(x^2-4xy-2y^2\right)dx+\left(y^2-4xy-2x^2\right)dy=0$.

18. Solve $\left(x+2y^3\right)\dfrac{dy}{dx}=y$.

19. Solve $xdy=[y-x\cos^2(y/x)]dx$

20. Solve (y-x-4)dx = (y+x-2)dy

21. Solve $\dfrac{dy}{dx}+\dfrac{x^3+3xy^2}{y^3+3x^2y}=0$

22. Solve $(x^2+y^2+x)dx+xydy=0$

23. Solve $(x+\tan y)dy=\sin 2y dx$

24. Find the orthogonal trajectories of the family of curves

$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2+\lambda}=1$ ('λ' being the parameter).

25. Show that the family of parabolas $y^2=4a(x+a)$ is self orthogonal.

26. Find the orthogonal trajectory of the cardiods $r=a\left(1-\cos\theta\right)$, using the differential equation method .

27. Find the orthogonal trajectories of the family of curve $r^n\cos n\theta=a^n$.

MODULE - V

LINEAR ALGEBRA

1. Investigate the value of λ and μ so that the equations $2x+3y+5z=9, 7x+3y-2z=8, 2x+3y+\lambda z=\mu$ have i) unique solution ii) no solution iii) infinite number of solutions.
2. For what values of λ and μ, the following simultaneous equations have i) a unique solution ii) no solution iii) an infinite number of solutions?
   $x+y+z=6$; $x+2y+3z=10$; $x+2y+\lambda z=\mu$
3. Find the rank of the matrix $\begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 92 & 93 & 94 & 95 & 96 \\ 93 & 94 & 95 & 96 & 97 \\ 94 & 95 & 96 & 97 & 98 \\ 95 & 96 & 97 & 98 & 99 \end{bmatrix}$.
4. Find the rank of the matrix $A=\begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 1 & 2 & 3 & 4 \end{bmatrix}$.
5. Using elementary row transformation find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.
6. Test for consistency and solve the system of equations
   $x+4+3z=0$, $x-y+z=0$, $2x-y+3z=0$
7. Test for consistency and solve the system of equations
   $$5x+y+3z=20$$
   $$2x+5y+2z=18$$
   $$3x+2y+z=14$$
8. Test for consistency and solve the system of equations
   $$x+y+z=1$$
   $$x+2y+3z=4$$
   $$x+3y+5z=7$$
   $$x+4y+7z=10$$

9. Solve following system of equations by Gauss elimination method

$$x + 2y + z = 3$$
$$2x + 3y + 3z = 10$$
$$3x - y + 2z = 13$$

10. Solve following system of equations by Gauss elimination method

$$2x + 3y - z = 5$$
$$4x + 4y - 3z = 3$$
$$2x - 3y + 2z = 2$$

11. Applying Gauss Jordan method solve $2x+3y-z=5$, $4x+4y-3z=3$, $2x-3y+2z=2$
12. Solve ,using Gauss Jordan method $x+y+z=9$, $x-2y+3z=8$, $2x+y-z=3$
13. Show that the transformation given below

$y_1 = 2x_1 + x_2 + x_3, y_2 = x_1 + x_2 + 2x_3, y_3 = x_1 - 2x_3,$ is regular and find the inverse transformation

14. Find the characteristics equation and eigen vector of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.
15. Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form using characteristics equation.
16. Find the matrix P which diagonalizes the matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{bmatrix}$
17. Find the eigen values and eigen vector corresponding to the largest eigen value of the

matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

18. If $P=\begin{bmatrix}-1 & 1 & 1\\ 0 & -1 & 2\\ 1 & 1 & 1\end{bmatrix}$ is a modal matrix of the matrix $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$ and inverses of P is $P^{-1}=\begin{bmatrix}-3 & 0 & 3\\ 2 & -2 & 2\\ 1 & 2 & 1\end{bmatrix}$, then transform A in to diagonal form and hence find A^4 .

19. Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2zy$ to the canonical form.

20. Reduce the quadratic form $x_1^2+3x_2^2+3x_3^2-2x_2x_3$ in to canonical form by an appropriate orthogonal transformation which transforms $x_1\ x_2\ x_3$ in terms of new variables $y_1\ y_2\ y_3$.

21. Find the nature of the quadratic forms for which corresponding eigen values of the corresponding matrices are given as

Matrix	Eigen values
A	2,3,4
B	-3,-4,-5
C	0,3,6
D	0,-3,-4
E	-2,3,4

22. *Use rayleigh's power method to find the largest eigen value and the corresponding eigen vector of the following matrices.*

$$\begin{bmatrix}1 & 6 & 1\\ 1 & 2 & 0\\ 0 & 0 & 2\end{bmatrix} \qquad (\text{Take } X_0 = [1, 0, 0]')$$

$$\begin{bmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 2 & 1 & 10 \end{bmatrix} \quad (\text{Take } X_0 = [0, 0, 1]')$$

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