QUESTION PAPER

MODULE I
DIFFERENTIAL CALCULUS-I

1) If y=cos(mlogx), prove that $x^2 y_{n+2} - (2n+1)xy_{n+1} + (m^2+n^2)y_n = 0$ (Jan 2015)

2) If $y^{1/m} + y^{-1/m} = 2x$, or $y = \left[x+\sqrt{x^2-1}\right]^m$ or $y = \left[x-\sqrt{x^2-1}\right]^m$
   Show that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$ (July 2015)

3) Find the pedal equation for the curve
   $$r^m = a^m \sin m\theta + b^m \cos m\theta$$ (July 2015)

4) Find the angle of intersection between the curves
   $r = a\log\theta$ and $r = a/\log\theta$ (Jan 2015)

5) Find the nth derivative of $\sin^3 x \cos^3 x$ (July 2015)

6) Find the radius of curvature of the curve
   $x = a\log(\sec t + \tan t)$, $y = a\sec t$ (July 2015)

7) Show that the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ (July 2015)

8) If $y = \sin\log(x^2+2x+1)$ *provethat* $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_n + (n^2+4)y_n = 0$

   (Jan 2014, Jan 2015)

9) Find the radius of curvature of $a^2 y = x^3 - a^3$ at the point
   where the curve cuts x-axis. (July 2014)

10) Prove that the radius of curvature ρ at any point (x,y) on the curve $\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1$ is given by $\rho=\frac{2\left(ax+by\right)^{3/2}}{ab}$. (Jan 2014)

11) Find the radius of curvature at any point t of the curve $x = a\left(cost + \frac{logtant}{2}\right)$, $y = asint.$ (July 2014)

12) Find the pedal equation of the curve $r^m = a^m \cos m\theta$ (Jan 2014 ,June 2013)

13) Find the radius of curvature for the curve $y^2=\frac{a^2(a-x)}{x}$, where the curve meets the x-axis (Jan 2014)

14) If x = sint and y = sinpt Prove that

$$(1-x^2)y_{n+2}-(2n+1)\,\mathrm{x}\,y_{n+1}+\left(p^2-n^2\right)y_n=0$$ (July2014)

MODULE II

DIFFERENTIAL CALCULUS-II

1) Obtain a Maclaurin's series for $f(x)=\tan x$ up to the term containing x^5. (July 2015)

2) Find first four non zero terms in the expansion of $f(\mathrm{x})=\frac{x}{e^{x-1}}$ (Jan 2015)

3) If $\cos u=\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-\frac{\cot u}{2}$. (Jan 2015)

4) If $\log u=\frac{x^3+y^3}{3x+4y}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u$. (July 2015)

5) Find $\frac{\partial\left(u,v,w\right)}{\partial\left(x,y,z\right)}$ where $u=x^2+\mathrm{y}^2+\mathrm{z}^2, v=xy+yz+zx, w=x+y+z$ (Jan 2015)

6) If $u=\frac{yz}{x}, v=\frac{zx}{y}, w=\frac{xy}{z}$, Show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=4$ (July2014, July 2015)

7) Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$ (Jan 2015)

8) Evaluate $\lim_{x\to 0}\left\{\frac{e^x \sin x - x - x^2}{x^2 + x\log x(1-x)}\right\}$ (July 2015)

9) If $z = f(x,y)$,where $x = r\cos\theta, y = r\sin\theta$ show that (Jan 2015)

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

10) Expand $\sqrt{1+\sin 2x}$ by Maclaurian series. (July2014)

11) Expand $f(x) = \sin(e^x - 1)$ in powers of 'x' upto the terms containing x^4. (June2014)

12) Expand *tanx* in powers of $(x - \pi/4)$ upto third degree term. (June2013)

13) Evaluate $\underset{x\to 0}{Lt}\left(\frac{1}{x} - \cot x\right)$. (June 2014)

14) Evaluate: i) $\underset{x\to 0}{Lim}\frac{\sin x \sin^{-1} x}{x^2}$ ii) $\underset{x\to 0}{Lim}\left(\frac{2^x + 3^x + 4^x}{3}\right)^{1/x}$. (Jan 2014)

15) Evaluate $\underset{x\to 0}{Lim}\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$. (Jan 2013)

16) Evaluate $\lim_{x\to 0}[\frac{a^x+b^x+c^x+d^x}{4}]^{1/x}$ (July 2014)

17) Evaluate $\lim_{x\to\infty}\left(\frac{ax+1}{ax-1}\right)^x$ (Jan 2014)

18) *If* $sinu = \frac{x^2y^2}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3tanu$. (June 2013)

19) If $u = \log\left(x^3 + y^3 + z^3 - 3xy\right)$ show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$ (Jan 2014)

20) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, find $x^2 \frac{\partial u}{\partial x}$. (July 2014)

21)) If $x = r sin\theta cos\varphi, y = r sin\theta sin\varphi, z = r cos\theta,$ find the Jacobian of (x, y, z) with respect to r, θ, φ.

(July 2014)

MODULE III
VECTOR CALCULUS

1) A particle moves on the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5,$ where t is the time. Find the components of velocity and acceleration at time t=1 in the direction $\hat{i} + 3\hat{j} + 2\hat{k}.$ (Jan2015)

2) Find the constants a,b,c such that the vector $\vec{F} = (x + y + az)\hat{i} + (x + cy + 2z)\hat{k} + (bx + 2y - z)\hat{j}$ is irrotational. (July 2015)

3) Prove that 1. $\nabla r^n = nr^{n-2}\vec{r},$ (July 2015)

4) Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = grad\left(x^3 + y^3 + z^3 - 3xyz\right)$ (July 2015)

5) If $\phi = x^2 + y^2 + z^2$ and $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k},$ then find $grad\phi, div\vec{F}, curl\vec{F}$. (July 2014)

6) Prove that $divCurlA = \nabla.\nabla \times A = 0$. (July 2013, July 2015)

7) *If* $\vec{V} = \vec{\omega} \times \vec{r}$ prove that $curl\vec{V} = 2\vec{\omega}$ *where* $\vec{\omega}$ *is a* constant vector. (July 2015)

8) *If* $\vec{r} = xi + yj + zk$ *and* $|\vec{r}| = r$, Find grad div$\left(\frac{\vec{r}}{r}\right)$. (Jan 2015)

9) Prove that $\nabla.\phi\vec{A} = \nabla\phi.\vec{A} + \phi\left(\nabla.\vec{A}\right)$. (July2014)

10) If $\vec{F} = 2xy^3z^4\,\vec{i} + 3x^2y^2z^4\,\vec{j} + 4x^2y^3z^3\,\vec{k}$, find $i)\ \left(\nabla.\vec{F}\right)$ $ii)\ \left(\nabla\times\vec{F}\right)$.

(Jan 2014)

11) Find the constants a,b,c so that the vector function

$\vec{F} = \left(x+2y+az\right)\vec{i} + \left(bx-3y-z\right)\vec{j} + \left(4x+y+2z\right)\vec{k}$ is irrotational .

(July 2014)

12) Show that the vector field $F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational and find its scalar potential. (July2014)

13) $If\ \vec{F} = \left(x+y+1\right)\vec{i} + \vec{j} - \left(x+y\right)\vec{k}\ , show\,that\ \vec{F}.curl\,\vec{F} = 0$

(Jan 2014)

14) Find the constant 'a' and 'b' such that $\vec{F} = (\mathrm{axy} + z^3)i + (3x^2 - z)j + (\mathrm{b}\,xz^2 - y)k$ is irrotational. Also find a scalar function ϕ *such that* $\vec{F} = \nabla\phi$ (Jan 2014,July 2014)

MODULE IV

INTEGRAL CALCULUS

1) Obtain the reduction formula of the integral $\int \cos^n x dx$. (Jan2015,July 2015)

2) Evaluate $\int_0^{\pi} x\sin^2 x\cos^4 x\,dx$ (Jan 2015)

3) Evaluate $\int_0^{2a} x^2(\sqrt{2ax-x^2})\,dx$. (July 2015 ,Jan2014,July 2014)

4) Find the orthogonal trajectories of the family of curves

$\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$ ('λ' being the parameter). (July 2015)

5) $\left(1+2xy\cos x^2-2xy\right)dx+\left(\sin x^2-x^2\right)dy=0$ (July 2015)

6) Solve $\left(xy^3+y\right)dx+2\left(x^2y^2+x+y^4\right)dy=0$ (J*an* 2015)

7) Evaluate $\int_0^{\pi/2} \sin^7\theta\cos^6\theta d\theta$. (June 2013)

8) Derive the reduction formula for $\int_0^{\pi/2} \cos^n x dx$. (July 2014)

9) Solve $\frac{dy}{dx}+xsin2y=x^3\cos^2 y$. (June 2013, July 2014)

10) Solve $\left(1+x^2\right)dx+\left(x-e^{-\tan^{-1}y}\right)dy=0$. (Jan 2014,June2013)

11) Solve $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xye^{xy^2}-3y^2\right)dy=0$. **(Jan 2014)**

12) Solve $\left(x^2-4xy-2y^2\right)dx+\left(y^2-4xy-2x^2\right)dy=0$. (June2013)

13) Solve $\left(x+2y^3\right)\frac{dy}{dx}=y$. (July 2015)

14) Find the orthogonal trajectories of the family of curve $r^n\cos n\theta=a^n$. (June 2013)

15) Find the orthogonal trajectory of the cardioids $r=a\left(1-\cos\theta\right)$, using the differential equation method . (Jan 2014)

16) Solve $\frac{dy}{dx}+xsin2y=x^3cos^2$y. (Jan 2015)

17) Find the orthogonal trajectory of the system of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$e λ is the parameter. (Jan 2015)

MODULE V
LINEAR ALGEBRA-I

1) Solve the following system of equations by Gauss's elimination method: (July 2015)

$$5x_1+x_2+x_3+x_4=4$$
$$x_1+7x_2+x_3+x_4=12$$
$$x_1+x_2+6x_3+x_4=-5$$
$$x_1+x_2+x_3+x_4=-6$$

2) Using Rayleigh's power method to find the largest Eigen value and the corresponding Eigen vector of the matrix.

(July 2015)

$$A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix} \qquad X^{(0)}=[1,0,0]$$

3) Find the matrix P which diagonalizes the matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{bmatrix}$ (Jan 2015)

4) Show that the transformation $y_1=x_1+2x_2+5x_3$, $y_2=2x_1+4x_2+11x_3$, $y_3=-x_2+2x_3$ is regular, write down the inverse transformation.

(Jan 2015)

5) Reduce the quadratic form $x^2+5y^2+z^2+2yz+6xz+2xy$ to the canonical form and specify the matrix of transformation.

(Jan2015,July 2014)

6) Find the rank of the matrix $\begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 92 & 93 & 94 & 95 & 96 \\ 93 & 94 & 95 & 96 & 97 \\ 94 & 95 & 96 & 97 & 98 \\ 95 & 96 & 97 & 98 & 99 \end{bmatrix}$. (July 2014)

7) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 1 & 2 & 3 & 4 \end{bmatrix}$. (June 2013)

8) Investigate the value of λ and μ so that the equations $2x+3y+5z=9, 7x+3y-2z=8, 2x+3y+\lambda z=\mu$ have i)unique solution ii) no solution iii) infinite number of solutions. (Jan 2015)

9) Using elementary row transformation find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

(June2014)

10) For what values of λ and μ, the following simultaneous equations have i) a unique solution i

ii) no solution iii) an infinite number of solutions?

$x+y+z=6$; $x+2y+3z=10$; $x+2y+\lambda z=\mu$ (July 2014)

11) Applying Gauss Elimination method solve

$2x+3y-z=5$, $4x+4y-3z=3$, $2x-3y+2z=2$

(Jan2015, July 2014)

12) Find the rank of the matrix A= $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ (Jan 2015)

13) Solve the system of equations by Gauss siedel method:

2x+5y+7z=52, 2x+y-z=0, x+y+z=9. (June2014)

14) Show that the transformation given below $y_1=2x_1+x_2+x_3, y_2=x_1+x_2+2x_3, y_3=x_1-2x_3,$ is regular and find the inverse transformation. (July2014)

15) Find the characteristics equation and eigen vector of the matrix $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.

(June 2013)

16) Reduce the matrix $A=\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form using characteristics equation.

` (July 2014)

17) Find the eigen values and eigen vector corresponding to the largest eigen value of the matrix

$A=\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (Jan 2015)

18) If $P=\begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ is a modal matrix of the matrix $A=\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ and inverses of P is

$P^{-1}=\begin{bmatrix} -0.5 & 0 & 0.5 \\ 0.33 & -0.33 & 0.33 \\ 0.16 & 0.33 & 0.16 \end{bmatrix}$, then transform A in to diagonal form and hence find A^4 .

(July 2014)

19) Reduce the quadratic form $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ in to canonical form by an appropriate orthogonal transformation which transforms $x_1\ x_2\ x_3$ in terms of new variables $y_1\ y_2\ y_3$.

(Jan 2015)

20) Find the nature of the quadratic forms for which corresponding eigen values of the corresponding matrices are given as

(Jan 2015)

Matrix	Eigen values
A	2,3,4
B	-3,-4,-5
C	0,3,6
D	0,-3,-4
E	-2,3,4

21) Find the Eigen values and the corresponding Eigen vectors of the matrix

(Jan 2014)

$$A=\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

22) Reduce the quadratic form $x^2+5y^2+z^2+2yz+6xz+2xy$ to the canonical form and specify the matrix of transformation.

(Jan 2014,July 2014)