Reg. No. :

Name :

SY-51

SECOND YEAR HIGHER SECONDARY EXAMINATION, MARCH 2020

Part – III

MATHEMATICS (COMMERCE)

Time : 2½ Hours

Maximum : 80 Scores

Cool-off time : 15 Minutes

General Instructions to Candidates :

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer any 6 questions from 1 to 8. Each carries 3 scores. **($6 \times 3 = 18$)**

1. (i) Form the 2×2 matrix $A = [a_{ij}]$, where $a_{ij} = i - j$ **(2)**

   (ii) Find : A' **(1)**

2. (i) The principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is **(1)**

   (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

   (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

   (ii) Find the sum :

   (a) $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$ **(1)**

   (b) $\cos^{-1}\left(\frac{-1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$ **(1)**

3. (i) If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ are two vectors, then find $|\vec{a}|$ and $|\vec{b}|$ **(1)**

   (ii) Find $\vec{a} \cdot \vec{b}$ **(1)**

   (iii) Find the angle between $\vec{a}$ and $\vec{b}$. **(1)**

4. (i) Find the value of x if $\tan^{-1} x = \frac{\pi}{4}$ **(1)**

   (ii) Show that

   $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$ **(2)**

5. The cost function for the production of x units of an item is $c(x) = 12x^2 + 5x + 6$. Find the marginal cost when 500 items are produced. **(3)**

6. Integrate the following w.r.t. x.

(i) $\frac{1}{1+x^2}$ **(1)**

(ii) $\frac{e^{\tan^{-1} x}}{1+x^2}$ **(2)**

7. (i) The number of straight lines passes through (1, 2, 1) and parallel to the vector $2\hat{i} + 3\hat{j} + \hat{k}$ is _____.

(a) one (b) infinity

(c) two (d) none **(1)**

(ii) Find the equation of a line passing through the point (1, 2, 1) and parallel to $2\hat{i} + 3\hat{j} + \hat{k}$ in both vector and Cartesian forms. **(2)**

8. Find the probability of getting the number 5 exactly twice in 7 throws of a die. **(3)**

Answer any 8 questions from 9 to 18. Each carries 4 scores. **($8 \times 4 = 32$)**

9. (i) Express the matrix $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. **(2)**

(ii) If the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a skew symmetric matrix, find the value of $\frac{a+b}{c+d}$. **(2)**

10. (i) Find the value of k if the function given by

$$f(x) = \begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases} \text{ is}$$

continuous on R. **(2)**

(ii) Show that the function $g(x) = \cos(x^2)$ is continuous. **(2)**

11. (i) What is the value of f(3) if $f(x) = 3x^2 + x - 3$? **(1)**

(ii) Find the product :

$f'(3) \cdot \Delta x$ if $\Delta x = 0.02$

Using this find the approximate value of f(3.02) **(3)**

12. (i) If $\int_a^b f(x)\,dx = k \int_b^a f(x)\,dx$, then k is **(1)**

(a) 1 (b) $\frac{1}{2}$

(c) –1 (d) 2

(ii) Find $\int_0^5 e^x\,dx$. **(1)**

(iii) Evaluate $\int_0^{\pi/4} e^x (\sin x + \cos x)\,dx$ **(2)**

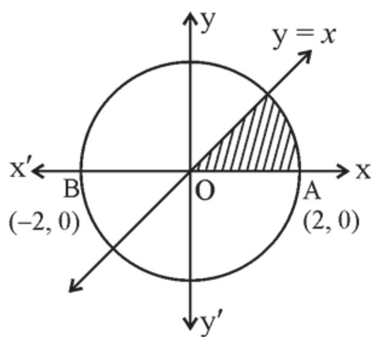
13. Given that $f(x) = x^3 - 3x + 3$.

(i) Find $f'(x)$ **(1)**

(ii) Verify Mean Value Theorem for $f(x) = x^3 - 3x + 3$ in the interval $[-1, 1]$ **(2)**

(iii) Find all $c \in [-1, 1]$ for which $f'(c) = 0$. **(1)**

14. In the figure, origin is the centre of the circle and $y = x$ is a straight line. Find the area of the shaded region. **(4)**



15. (i) Write the order and degree of the differential equation :

$$x\frac{dy}{dx} = x + y$$ **(1)**

(ii) Solve : $x\frac{dy}{dx} = x + y$ **(3)**

16. (i) Find the Cartesian equation of the plane passing through the points (1, 1, 0), (1, 2, 1) and (–2, 2, –1) **(2)**

(ii) Write the x, y, z intercepts of the plane given above. **(2)**

17. A pharmaceutical company produces 2 types of medicines A and B which requires 2 ingredients C and D. The requirement to produce one bottle (50 ml) each of A and B are given below :

	A	B	Max. Availability of C and D
C	20 ml	40 ml	3000 ml
D	30 ml	10 ml	5000 ml

To produce maximum number of bottles of medicines A and B, formulate the problem as an LPP. **(4)**

(No graph or solution required.)

18. Given that A and B are two independent events and $P(A) = \frac{2}{10}$; $P(B) = \frac{4}{10}$.

Then find,

(i) $P(A / B) \times P(B / A)$ **(2)**

(ii) $P(A \cup B)$ **(2)**

Answer any 5 questions from 19 to 25. Each carries 6 scores. **(5 × 6 = 30)**

19. Let f and g be two functions on R given by $f(x) = 3 + 4x$; $g(x) = x^2$

(i) Show that f is one-one. **(2)**

(ii) Show that g is many-one. **(1)**

(iii) Considering the domain and range of both f and g as R^+, find $f \circ g\,(x)$ and $g \circ f\,(x)$. **(3)**

20. Let $A = \begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix}$. Then,

(i) Evaluate det (A) **(1)**

(ii) Show that det (3A) = 9.det (A) **(3)**

(iii) Find x if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ **(2)**

21. (i) Differentiate : $y = \cos(x^2)$ **(2)**

(ii) If $y = e^x (x^2 - 1)$, show that

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{2y}{x-1}$$ **(4)**

22. (i) Find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ **(4)**

(ii) Solve the system of equations

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

using the above matrix. **(2)**

23. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ are two vectors,

(i) Find $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ **(2)**

(ii) Find $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$ **(2)**

(iii) Find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$. **(2)**

24. Consider the following L.P.P.

Max : $z = 3x + 2y$

Subject to

$x + y \leq 10, 0 \leq y \leq 8, x \geq 0$

(i) Draw the feasible region of the given L.P.P. **(4)**

(ii) Find the solution of the L.P.P. **(2)**

25. A random variable X has the following probability distribution :

X :	0	1	2	3	4	5	6
P(X) :	k	k + 0.1	0	2k	2k	0.2	k

(i) Find k **(1)**

(ii) Find (a) $P(X \leq 2)$ (b) $P(X > 3)$ **(2)**

(iii) Find the mean of the random variable X. **(3)**