Reg. No. :

Name :

SY-27

SECOND YEAR HIGHER SECONDARY EXAMINATION, MARCH 2020

Part – III

MATHEMATICS (SCIENCE)

Maximum : 80 Scores

Time : 2½ Hours

Cool-off time : 15 Minutes

General Instructions to Candidates :

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer any six questions from 1 to 8. Each carry 3 scores. **$(6 \times 3 = 18)$**

1. (i) Let R be a relation in the set $\mathbb{N}$ of natural numbers given by $R = \{(a, b) : a = b - 2\}$. Choose the correct answer. **(1)**

   (a) $(2, 3) \in R$ (b) $(3, 8) \in R$

   (c) $(6, 8) \in R$ (d) $(8, 7) \in R$

   (ii) Let $*$ be a binary operation defined on the set $\mathbb{Z}$ of integers as $a * b = a + b + 1$. Then find the identity element. **(2)**

2. (i) Write two non-zero matrices A and B for which $AB = 0$. **(1)**

   (ii) Express $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ as the sum of a symmetric matrix and a skew symmetric matrix. **(2)**

3. Using properties of determinates, prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$. **(3)**

4. (i) Which among the following is not true :

   (a) A polynomial function is always continuous.

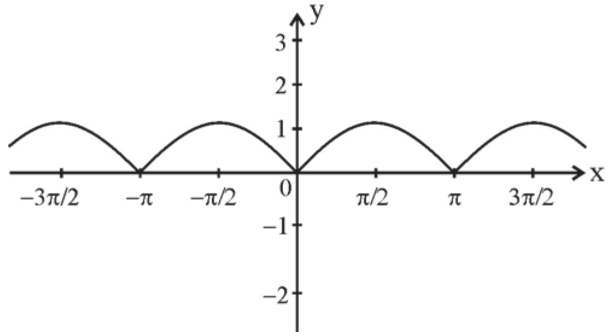
   (b) A continuous function is always differentiable.

   (c) A differentiable function is always continuous.

   (d) $\log x$ is continuous for all x greater than zero. **(1)**

   (ii) Find $\frac{dy}{dx}$, if $x^2 + y^2 + xy = 100$. **(2)**

5. (i) Identify the following function. **(1)**



(a) $\sin x$ (b) $|\sin x|$

(c) $\sin |x|$ (d) $\cos x$

(ii) Is the above function differentiable ? Why ? **(1)**

(iii) Find derivative of $y = \sqrt{\tan x}$ **(1)**

6. (i) The slope of the tangent to the curve $y = e^{2x}$ at (0, 1) is **(1)**

(a) 1 (b) 2

(c) 0 (d) –1

(ii) Find the equation of a line perpendicular to the above tangent (tangent obtained in part (i)) and passing through (2, 3). **(2)**

7. (i) The general solution of a differential equation contains 3 arbitrary constants. Then what is the order of the differential equation ? **(1)**

(a) 2 (b) 3

(c) 0 (d) 1

(ii) Check whether $y = e^{-3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$. **(2)**

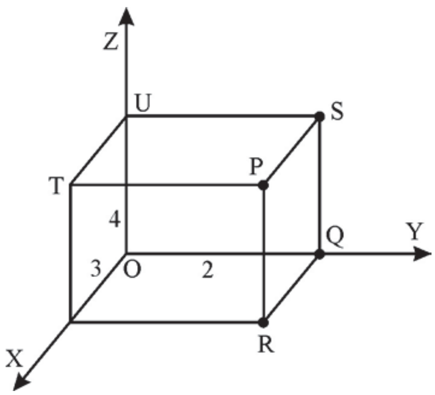
8. Consider the following figure :

(i) The equation of the plane PRQS is **(1)**

(a) y = 0 (b) y = 2

(c) z = 4 (d) $x = 3$

(ii) Find the equation of the plane through the intersection of the planes PRQS and PSUT and the point (2, 1, 2). **(2)**



Answer any 8 questions from 9 to 18. Each carry 4 scores. **(8 × 4 = 32)**

9. Let A = $\mathbb{R} - \{3\}$ and B = $\mathbb{R} - \{1\}$. Consider the function f : A → B defined by $f(x) = \dfrac{x-2}{x-3}$.

(i) Is f one-one and onto ? Justify your answer. **(2)**

(ii) Is it invertible ? Why ? **(1)**

(iii) If invertible, find inverse of f(x). **(1)**

10. (i) If $xy < 1$, $\tan^{-1} x + \tan^{-1} y$ = ________. **(1)**

(a) $\tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$ (b) $\tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$

(c) $\dfrac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$ (d) $\dfrac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$

(ii) Solve $\tan^{-1} 2x + \tan^{-1} 3x = \dfrac{\pi}{4}$. **(3)**

11. (i) Find $\frac{dy}{dx}$ if $y = x^x + x^{\sin x}$. **(3)**

(ii) If $y = x \cos x$, find $\frac{d^2y}{dx^2}$. **(1)**

12. (i) $\int \frac{f(x)}{\tan x} dx = \log|\tan x| + c$. Then f(x) is **(1)**

(a) $\cot x$ (b) $\sec^2 x$

(c) $\operatorname{cosec}^2 x$ (d) $\cot^2 x$

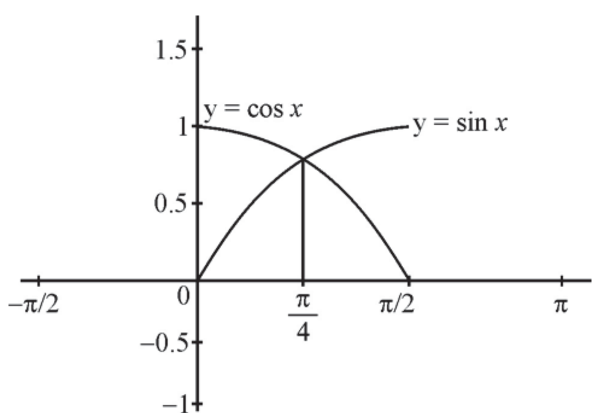
(ii) If $\frac{d(f(x))}{dx} = 4x^3 - \frac{3}{x^4}$; $x \neq 0$. Given that f(2) = 0. Find f(x). **(3)**

13. (i) Area bounded by the curve y = f(x), x-axis and the lines x = a and x = b is **(1)**

(a) $\int_a^b x\, dy$ (b) $\int_a^b y\, dx$

(c) $\int_a^b x^2\, dy$ (d) $\int_a^b y^2\, dx$

(ii) From the following figure, find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$ and x-axis as x varies from 0 to $\frac{\pi}{2}$. **(3)**



14. (i) Form the differential equation corresponding to the curve y = mx. **(2)**

(ii) Solve $\frac{dy}{dx}+\frac{y}{x}=x^2$. **(2)**

15. Find a unit vector perpendicular to the plane ABC where A, B, C are points (1, 1, 2), (2, 3, 5) and (1, 5, 5). **(4)**

16. The Cartesian equation of two lines are

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1} \text{ and } \frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}.$$

(i) Write the vector equations. **(1)**

(ii) Find the shortest distance between these two lines. **(3)**

17. (i) If a plane intersects the co-ordinate axes at a, b, c respectively, write the equation of the plane. **(1)**

(ii) Find the distance of the plane obtained in part (i) from the origin. **(1)**

(iii) Find the Vector and Cartesian equations of the plane passing through (1, 0, –2) and normal to the plane is i + j – k. **(2)**

18. Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6 find

(i) P(A and B) **(1)**

(ii) P(A and not B) **(1)**

(iii) P(A or B) **(1)**

(iv) P(neither A nor B) **(1)**

Answer any 5 questions from 19 to 25. Each carry 6 scores. **(5 × 6 = 30)**

19. (i) Let $A = [a_{ij}]_{2 \times 3}$; where $a_{ij} = i + j$. Construct A. **(2)**

(ii) Find AA' and hence prove that AA' is symmetric. **(2)**

(iii) For any square matrix A, prove that A + A' is symmetric. **(2)**

20. (i) If A is a skew symmetric matrix of order 3. Then prove that its determinant is zero (Without using example). **(2)**

(ii) Given that $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & 5 \end{bmatrix}$ is a singular matrix. Find the value of x. **(2)**

(iii) Given A and B are square matrices of order 2 such that | A | = –1, | B | = 3. Find | 3AB | **(2)**

21. (i) Find the intervals in which the function $f(x) = x^2 + 2x - 5$ strictly increasing or decreasing. **(2)**

(ii) Find the equation of tangent and normal for the curve $y = x^3$ at (1, 1). **(2)**

(iii) Find local maximum and local minimum if any for the function

$h(x) = \sin x + \cos x,\ 0 < x < \frac{\pi}{2}$. **(2)**

22. Integrate :

(i) $\int \frac{dx}{1+\frac{x^2}{4}}$ **(2)**

(ii) $\int \frac{x}{(x-1)(x-2)} dx$ **(2)**

(iii) $\int_0^{\frac{\pi}{2}} x \cos x \, dx$ **(2)**

23. (i) If $\bar{a}, \bar{b}, \bar{c}$ are three coplanar vectors, then $[\bar{a}\,\bar{b}\,\bar{c}]$ is

(a) 1 (b) 0

(c) –1 (d) not defined **(1)**

(ii) If $|\bar{a}| = 2$, $|\bar{b}| = 3$ and θ is the angle between $\bar{a}$ and $\bar{b}$. Then maximum value of $\bar{a} \cdot \bar{b}$ occurs when θ = ________.

(a) $\frac{\pi}{2}$ (b) π

(c) 0 (d) $\frac{\pi}{4}$ **(1)**

(iii) If $\bar{b} = 2i + j - k$, $\bar{c} = i + 3k$ and $\bar{a}$ is a unit vector. Find the maximum value of Scalar triple product $[\bar{a}\,\bar{b}\,\bar{c}]$. **(4)**

24. Solve the linear programming problem graphically.

Max : $Z = 3x + 2y$

Subject to : $x + 2y \leq 10$

$3x + y \leq 15$

$x \geq 0,\ y \geq 0$ **(6)**

25. The probability distribution of a random variable X is given in the following table :

X	0	1	2	3	4
P(X)	0.1	k	2k	2k	k

(i) Find k. **(1)**

(ii) Find the probability that X lies between 1 and 4. **(1)**

(iii) Find mean of X. **(2)**

(iv) Find variance of X. **(2)**