Solutions to Question Bank
Module-I
Modern physics and Quantum physics

1). Calculate the de-Broglie wavelength ofparticle of mass 0.65MeV/C^2 has a kinetic energy of 80eV. (Jan2013)

Given:
M= 0.65MeV/c^2=1.157x10^{-30} kg.
E=80eV
De Brogli wavelength $\lambda=h/p=h/\sqrt{2Em}=6.625\text{x}10^{-34}/\sqrt{2X80X9.1X10^{-31}}$
$\lambda=1.1218\text{x}10^{-10}$m

2). Set up Schrodinger time- independent wave equation.(Jan 2013)

Consider a particle of mass ‘m’ moving with velocity ‘v’. The de-Broglie wavelength ‘λ’ is

$$\lambda = \frac{h}{m\text{v}} = \frac{h}{P} \rightarrow (1)$$

Where ‘mv’ is the momentum of the particle.

The wave eqn is

$$\Psi = A\, e^{\,i(kx - \Omega t)} \rightarrow (2)$$

Where ‘A’ is a constant and ‘ω’ is the angular frequency of the wave.

Differentiating equation (2) with respect to ‘t’ twice

$$\frac{d^2\Psi}{dt^2} = -A\Omega^2 e^{\,i(kx-\Omega t)} = -\Omega^2\Psi \qquad \rightarrow (3)$$

The equation of a travelling wave is

$$\frac{d^2y}{dx^2} = \frac{1}{\text{v}^2}\frac{d^2y}{dt^2}$$

Where ‘y’ is the displacement and ‘v’ is the velocity.

Similarly for the de-Broglie wave associated with the particle

$$\frac{d^2\Psi}{dx^2} = \frac{1}{\mathrm{v}^2}\frac{d^2\Psi}{dt^2} \rightarrow (4)$$

where ‘ψ’ is the displacement at time ‘t’.

From eqns (3) & (4)

$$\frac{d^2\Psi}{dx^2} = -\frac{\Omega^2}{\mathrm{v}^2}\Psi$$

But ω = 2πυ and v =υ λ where ‘υ’ is the frequency and ‘λ’ is the wavelength.

$$\frac{d^2\Psi}{dx^2} = -\frac{4\Pi^2}{\Lambda^2}\Psi \text{ or } \frac{1}{\Lambda^2} = -\frac{1}{4\Pi^2\Psi}\frac{d^2\Psi}{dx^2} \rightarrow (5)$$

$$K.E = \frac{1}{2}m\mathrm{v}^2 = \frac{m^2\mathrm{v}^2}{2m} = \frac{P^2}{2m} \rightarrow (6)$$

$$= \frac{h^2}{2m\Lambda^2} \rightarrow (7)$$

Using eqn (5)

$$K.E = \frac{h^2}{2m}\left[-\frac{1}{4\Pi^2\Psi}\right]\frac{d^2\Psi}{dx^2} = -\frac{h^2}{8\Pi^2 m\Psi}\frac{d^2\Psi}{dx^2} \rightarrow (8)$$

Total Energy E = K.E + P.E

$$E = -\frac{h^2}{8\Pi^2 m\Psi}\frac{d^2\Psi}{dx^2} + V$$

$$E - V = -\frac{h^2}{8\Pi^2 m\Psi}\frac{d^2\Psi}{dx^2}$$

$$\frac{d^2\Psi}{dx^2} = -\frac{8\Pi^2 m}{h^2}(E - V)\Psi$$

$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 m}{h^2}(E - V)\Psi = 0}$$

This is the time independent Schrodinger wave equation

3).Using Heisenberg's uncertainty principle and prove that free electron does not exist inside the nucleus (Jan 2013)

According to the theory of relativity, the energy E of a particle is:

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1 - (v^2/c^2)}}$$

Where 'm_o' is the rest mass of the particle and 'm' is the mass when its velocity is 'v'.

$$\text{i.e. } E^2 = \frac{m_o^2 c^4}{1 - (v^2/c^2)} = \frac{m_o^2 c^6}{c^2 - v^2} \rightarrow (1)$$

If 'p' is the momentum of the particle:

$$\text{i.e. } p = mv = \frac{m_o v}{\sqrt{1 - (v^2/c^2)}}$$

$$p^2 = \frac{m_o^2 v^2 c^2}{c^2 - v^2}$$

Multiply by c^2

$$p^2c^2 = \frac{m_o^2 v^2 c^4}{c^2 - v^2} \rightarrow (2)$$

Subtracting (2) by (1) we have

$$E^2 - p^2c^2 = \frac{m_o^2 c^4 (c^2 - v^2)}{c^2 - v^2}$$

$$E^2 = p^2c^2 + m_o^2c^4 \rightarrow (3)$$

Heisenberg's uncertainty principle states that

$$\Delta x \,.\, \Delta P_x \geq \frac{h}{4\Pi} \rightarrow (4)$$

The diameter of the nucleus is of the order 10^{-14}m. If an electron is to exist inside the nucleus, the uncertainty in its position Δx must not exceed 10^{-14}m.

i.e. $\Delta x \leq 10^{-14}$m

The minimum uncertainty in the momentum

$$(\Delta P_x)_{\min} \geq \frac{h}{4\Pi\ (\Delta x)_{\max}} \geq \frac{6.63 \times 10^{-34}}{4\Pi \times 10^{-14}} \geq 0.5 \times 10^{-20} \text{ kg.m/s} \rightarrow (5)$$

By considering minimum uncertainty in the momentum of the electron

i.e., $(\Delta P_x)_{\min} \geq 0.5 \times 10^{-20}$ kg.m/s = p → (6)

Consider eqn (3)

$$E^2 = p^2c^2 + m_o^2c^4 = c^2(p^2+m_o^2c^2)$$

Where $m_o = 9.11 \times 10^{-31}$ kg

If the electron exists in the nucleus its energy must be

$$E^2 \geq (3 \times 10^8)^2[(0.5 \times 10^{-20})^2 + (9.11 \times 10^{-31})^2(3 \times 10^8)^2]$$

i.e. $E^2 \geq (3 \times 10^8)^2[0.25 \times 10^{-40} + 7.4629 \times 10^{-44}]$

Neglecting the second term as it is smaller by more than the 3 orders of the magnitude compared to first term.

Taking square roots on both sides and simplifying

$$E \geq 1.5 \times 10^{-12} \text{ J} \geq \frac{1.5 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ ev} \geq 9.4 \text{ Mev}$$

If an electron exists in the nucleus its energy must be greater than or equal to 9.4Mev. It is experimentally measured that the beta particles ejected from the nucleus during beta decay have energies of about 3 to 4 MeV. This shows that electrons cannot exist in the nucleus.

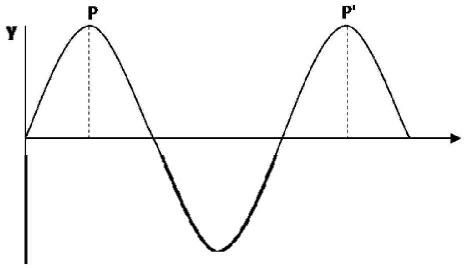
4).Derive the expression for group velocity on the basis of superposition of waves. Also obtain the relation between group velocity and phase velocity. (july 2013)

Phase velocity and group velocity:

A wave is represented by the equation:

$y = A\sin(\omega t - kx)$

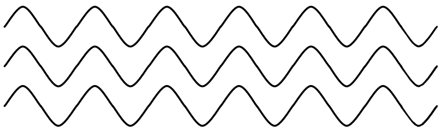
Where 'y' is the displacement along Y-axis at an instant t, 'ω' is the angular frequency, 'k' is propagation constant or wave number. 'x' is the displacement along x-axis at the instant 't'.



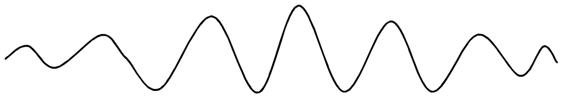
If 'p' is the point on a progressive wave, then it is the representative point for a particular phase of the wave, the velocity with which it is propagated owing to the motion of the wave is called *phase velocity.*

The phase velocity of a wave is given by $v_{phase} = (\omega / k)$.

Group velocity:



Individual Waves



Amplitude variation after Superposition

A group of two or more waves, slightly differing in wavelengths are super imposed on each other. The resultant wave is a packet or wave group. The velocity with which the envelope enclosing a wave group is transported is called *Group Velocity*.

Let $y_1 = A\sin(\omega t - kx) \rightarrow (1)$ and $y_2 = A\sin[(\omega+\Delta\omega)t - (k+\Delta k)x] \rightarrow (2)$

The two waves having same amplitude & slightly different wavelength. Where y_1 & y_2 are the displacements at any instant‘t’, ‘A’ is common amplitude, ‘Δω’ & ‘Δk’ are difference in angular velocity and wave number are assumed to be small. ‘x’ is the common displacement at time ‘t’

By the principle of superposition

$$y = y_1 + y_2$$

$$y = A\sin(\omega t - kx) + A\sin\{(\omega+\Delta\omega)t - (k+\Delta k)x\}$$

But, $\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$

$$y = 2A\cos\left(\frac{\Delta\Omega}{2}t - \frac{\Delta k}{2}x\right)\sin\left(\frac{2\Omega+\Delta\Omega}{2}t - \frac{2k+\Delta k}{2}x\right)$$

Since Δω and Δk are small

$2\omega + \Delta\omega \approx 2\omega$ and $2k + \Delta k \approx 2k$.

$$\therefore\ y = 2A\cos\left(\frac{\Delta\Omega}{2}t - \frac{\Delta k}{2}x\right)\sin(\Omega t - kx) \rightarrow (3)$$

From equations (1) & (3) it is seen the amplitude becomes

$$2A\cos\left(\frac{\Delta\Omega}{2}t - \frac{\Delta k}{2}x\right)$$

The velocity with which the variation in amplitude is transmitted in the resultant wave is the group velocity.

$$v_{group} = \frac{(\Delta\Omega/2)}{(\Delta k/2)} = \frac{\Delta\Omega}{\Delta k}$$

In the limit $\frac{\Delta\Omega}{\Delta k} \rightarrow \frac{d\Omega}{dk}$

$$v_{group} = \frac{d\Omega}{dk}$$

__Relation between group velocity and phase velocity:__

The equations for group velocity and phase velocity are:

$$v_{group} = \frac{d\Omega}{dk} \rightarrow (1) \ \& v_{phase} = \frac{\Omega}{k} \rightarrow (2)$$

Where ‘ω’ is the angular frequency of the wave and ‘k’ is the wave number.

$$\omega = k\, v_{phase}$$

$$v_{group} = \frac{d\Omega}{dk} = \frac{d}{dk}(k v_{phase})$$

$$v_{group} = v_{phase} + k\frac{dv_{phase}}{dk}$$

$$v_{group} = v_{phase} + k\left(\frac{dv_{phase}}{d\Lambda}\right)\left(\frac{d\Lambda}{dk}\right) \rightarrow (3)$$

We have k = (2π/λ)

Differentiating $\frac{dk}{d\Lambda} = -\frac{2\Pi}{\Lambda^2}$ or $\frac{d\Lambda}{dk} = -\frac{\Lambda^2}{2\Pi}$

$$k\frac{d\Lambda}{dk} = \left(\frac{2\Pi}{\Lambda}\right)\left(-\frac{\Lambda^2}{2\Pi}\right) = -\Lambda$$

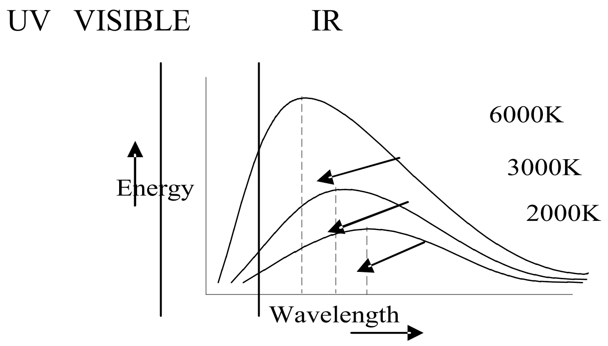
Using this in eqn (3)

$$v_{group} = v_{phase} - \lambda\frac{dv_{phase}}{d\Lambda}$$

This is the relation between group velocity and phase velocity

5)) Explain blackbody radiation spectrum on the basis of planks radiation law. (Dec2014)

A Blackbody is one which absorbs the entire radiation incident on it and emits all the absorbed radiation when it is more hot. A true blackbody does not exist practically. A blackbody designed by Wein has features very close to the true blackbody. Ferry has also constructed blackbody called Ferry's blackbody. A blackbody at a particular temperature found to emit a radiation of all possible wavelengths. It is a continuous spectrum starting from certain minimum wavelength to maximum wavelength. The maximum intensity corresponds to a particular wavelength. For different temperatures of the black body, there are different curves. As the temperature of the body increases, the wavelength corresponding to maximum intensity shifts towards lower wavelength side. The distribution of energy in black body radiation is shown in the following fig.



Wein's, Rayleigh-Jeans and Planck have given their explanations to account these observed experimental facts as follows:

Planck's Law:

Planck assumed that walls of the experimental blackbody consists larger number of electrical oscillators. Each oscillator vibrates with its own frequency.

i) Each oscillator has an energy given by integral multiple of hυ where h is Planck's constant & υ is the frequency of vibration.

$$E = nh\upsilon \text{ where } n = 1, 2, 3 \ldots \text{etc.}$$

ii) An oscillator may lose or gain energy by emitting or absorbing respectively a radiation of frequency υ where $\upsilon = \Delta E/h$, ΔE is difference in energies of the oscillator before and after the emission or absorption take place.

Planck derived the law which holds good for the entire spectrum of the blackbody radiation as

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{}{e^{h\nu/kT}-1}\right]d\lambda \quad \text{(since } \upsilon = c/\lambda\text{)} \qquad \xrightarrow{(1)}$$

This is Planck's Radiation Law.

Wein's law:The relation between the wavelength of emission and the temperature of the source is

$$U_\lambda \, d\lambda = C_1 \lambda^{-5} e^{-\left(\frac{C_2}{\lambda T}\right)} d\lambda$$

Where $U_\lambda d\lambda$ is the energy / unit volume in the range of wavelength λ and λ+dλ, C_1 and C_2 are constants.

This is called Wein's law of energy distribution in the black body radiation spectrum.

Drawbacks of Wein's law:

Wein's law holds good for the shorter wavelength region and high temperature of the source. It failed to explain gradual drop in intensity of radiation corresponding to longer wavelength greater than the peak value.

Rayleigh-Jeans Law:

Rayleigh derived an equation for the blackbody radiation on the basis of principle of equipartition of energy. The principle of equipartition of energy suggests that an average energy kT is assigned to each mode of vibration. The number of vibrations/unit volume whose wavelength is in the range of λ and λ+dλ is given by $8\pi\lambda^{-4}d\lambda$.

The energy/unit volume in the wavelength range λ and λ + dλ is

$U_\lambda d\,\lambda = 8\pi kT\lambda^{-4}d\lambda$

This is Rayleigh-Jeans equation. Accordingly energy radiated by the blackbody decreases with increasing wavelength.

Rayleigh-Jeans Law predicts to radiate all the energy at shorter wavelength side but it does not happen so. A black body radiates mainly in the infra-red or visible region of electromagnetic spectrum and intensity of radiation decreases down steeply for shorter wavelengths. Thus, the Rayleigh-Jeans Law fails to explain the lower wavelength side of the spectrum. This is referred to as ultra-violet Catastrophe.

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6). Obtain the solution of Schrödinger's time –independent wave equation when applied to a potential box of infinite height. (Dec2014)

Consider a particle of a mass 'm' free to move in one dimension along positive x-direction between x=0 to x=a. The potential energy outside this region is infinite and within the region is zero. The particle is in bound state. Such a configuration of potential in space is called infinite potential well. It is also called particle in a box. The Schrödinger equation outside the well is

$$\frac{d^2\Psi}{dx^2}+\frac{8\Pi^2 m}{h^2}(E-\infty)\Psi=0\rightarrow(1) \qquad \because V=\infty$$

For outside, the equation holds good if ψ = 0 & $|\psi|^2 = 0$. That is particle cannot be found outside the well and also at the walls

The Schrodinger's equation inside the well is:

$$\frac{d^2\Psi}{dx^2}+\frac{8\Pi^2 m}{h^2}E\Psi=0 \rightarrow(2) \qquad \because V=0$$

$$-\frac{h^2}{8\Pi^2 m}\frac{d^2\Psi}{dx^2}=E\Psi \rightarrow(3)$$

This is in the form $\hat{H}\psi = E\psi$

This is an Eigen-value equation.

Let $\frac{8\Pi^2 m}{h^2}E=k^2$ in eqn (2)

$$\frac{d^2\Psi}{dx^2}+k^2\Psi=0$$

The solution of this equation is:

$$\psi = C\cos kx + D\sin kx \rightarrow (4)$$

at $x = 0 \rightarrow \psi = 0$

$$0 = C\cos 0 + D\sin 0$$

$\therefore$ C = 0 Also $x = a \rightarrow \psi = 0$

$$0 = C\cos ka + D\sin ka$$

But C = 0

$\therefore D\sin ka = 0$ (5)

D≠0 (because the wave concept vanishes)

i.e. $ka = n\pi$ where $n = 0, 1, 2, 3, 4\ldots$ (quantum number)

$$k=\frac{n\Pi}{a}\rightarrow(6)$$

Using this in eqn (4)

$$\Psi_n = D\sin\frac{n\Pi}{a}x \rightarrow (7).$$

To find out the value of D, normalization of the wave function is to be done.

$$\int_0^a |\psi_n|^2 dx = 1 \rightarrow (8) \text{ using the values of } \psi\text{n from eqn } (7)$$

$$\int_0^a D^2\sin^2\frac{n\Pi}{a}xdx = 1$$

$$D^2\int_0^a \left[\frac{1-\cos(2n\Pi/a)x}{2}\right]dx = 1$$

$$\frac{D^2}{2}\left[\int_0^a dx - \int_0 \cos\frac{2n\Pi}{a}xdx\right] = 1$$

$$\frac{D^2}{2}\left[x - \frac{a}{2n\Pi}\sin\frac{2n\Pi}{a}x\right]_0^a = 1$$

$$\frac{D^2}{2}[a-0] = 1$$

$$\frac{D^2}{2}a = 1$$

$$D = \sqrt{\frac{2}{a}}$$

Hence the normalized wave functions of a particle in one dimensional infinite potential well is:

$$\Psi_n = \sqrt{\frac{2}{a}}\sin\frac{n\Pi}{a}x \rightarrow (9)$$

Energy Eigen values:

From Eq. 6 &2

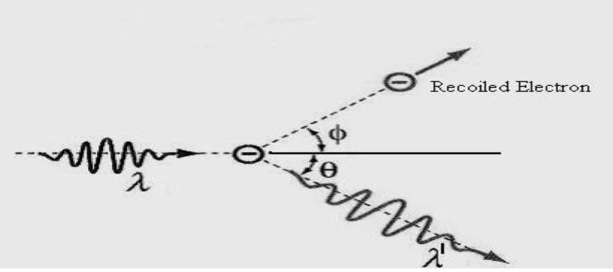
$$\frac{8\Pi^2}{m} \quad E = k^2 = \frac{n^2\Pi^2}{a^2}$$

$$E = \frac{n^2h^2}{8ma^2}$$

7). What is Compton Effect? Explain its Physical significance. (Dec2014)

The scattering of a photon by an electron is called as Compton effect or Compton scattering.

When a photon of wavelength ‘λ’ is scattered by an electron in the direction making an angle ‘θ’ with the direction of incidence, the wavelength of the scattered photon increases. Its wavelength is λ'. The electron recoils in a direction making an angle ‘φ’ with the incident direction of photon. The difference in the wavelength (λ'-λ) is called the Compton shift. Compton found that λ' is independent of the target material, but depends on the angle of scattering.



If λ is the wavelength of the incident photon, its energy E is given by E=hc/λ where ‘h’ is the Planck’s constant, ‘c’ is the velocity of light, ‘λ’ is the wavelength of the incident photon. If λ' is the wavelength of the scattered photon, its energy E' is given by

*Physical significance:*In Compton effect, the Compton shift is explained on the basis that the X-ray photon collide with the electron and during the collision, the energy exchange between the two occurs as though it is a kind of partical-partical collision. Thus it demonstrates the particle nature of X-ray which we know is electromagnetic waves, or in other words it signifies the particle nature of waves in general.

8). The position and momentum of electron with energy 0.5 Kev are determined .What is minimum percentage uncertainty in its momentum if the uncertainty in the measurement of position is 0.5A. (Dec2014)

Data: Δx = 0.5A

$$\Delta p \geq \frac{h}{4\Pi} \times \frac{1}{\Delta x}$$

$$\Delta p = \frac{6.625\times10^{-34}}{4\times3.14\times0.5\times10^{-10}} = 1.05 \text{ x } 10^{-24} \text{ Ns}$$

P =

P = = 1.207 x 10^{-23} Ns

% of uncertainty =

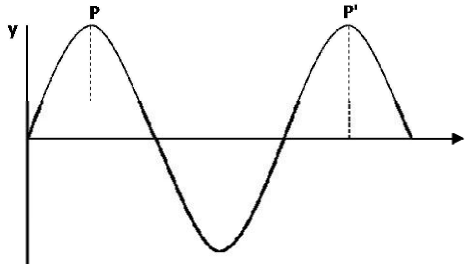
9). What is group velocity and phase velocity in wave motion? Obtain the relation between them. (Dec2014)

Phase velocity and group velocity:

A wave is represented by the equation:

$$y = A\sin(\omega t - kx)$$

Where ‘y’ is the displacement along Y-axis at an instant t, ‘ω’ is the angular frequency, ‘k’ is propagation constant or wave number. ‘x’ is the displacement along x-axis at the instant ‘t’.



If ‘p’ is the point on a progressive wave, then it is the representative point for a particular phase of the wave, the velocity with which it is propagated owing to the motion of the wave is called *phase velocity.*

The phase velocity of a wave is given by $v_{phase} = (\omega / k)$.

A group of two or more waves, slightly differing in wavelengths are super imposed on each other. The resultant wave is a packet or wave group. The velocity with which the envelope enclosing a wave group is transported is called *Group Velocity*.

$$v_{group} = \frac{d\Omega}{dk}$$

Relation between group velocity and phase velocity:

The equations for group velocity and phase velocity are:

$$v_{group} = \frac{d\Omega}{dk} \rightarrow (1) \; \& v_{phase} = \frac{\Omega}{k} \rightarrow (2)$$

Where ‘ω’ is the angular frequency of the wave and ‘k’ is the wave number.

$\omega = k\, v_{phase}$

$$v_{group} = \frac{d\Omega}{dk} = \frac{d}{dk}(k v_{phase})$$

$$v_{group} = v_{phase} + k\, \frac{dv_{phase}}{dk}$$

$$v_{group} = v_{phase} + k \left(\frac{dv_{phase}}{d\Lambda}\right)\left(\frac{d\Lambda}{\ }\right) \rightarrow (3)$$

We have $k = (2\pi/\lambda)$

Differentiating $\frac{dk}{d\Lambda} = -\frac{2\Pi}{\Lambda^2}$ or $\frac{d\Lambda}{dk} = -\frac{\Lambda^2}{2\Pi}$

$$k\frac{d\Lambda}{dk} = \left(\frac{2\Pi}{\Lambda}\right)\left(-\frac{\Lambda^2}{2\Pi}\right) = -\Lambda$$

Using this in eqn (3)

$$v_{group} = v_{phase} - \lambda\, \frac{dv_{phase}}{d\Lambda}$$

This is the relation between group velocity and phase velocity.

10). Setup time independent Schrödinger wave equation for free particle in one dimension.

(Dec2014)

Consider a particle of mass 'm' moving with velocity 'v'. The de-Broglie wavelength 'λ' is

$$\lambda = \frac{h}{mv} = \frac{h}{P} \rightarrow (1)$$

Where 'mv' is the momentum of the particle.

The wave eqn is

$$\Psi = A\, e^{i(kx - \Omega t)} \rightarrow (2)$$

Where 'A' is a constant and 'ω' is the angular frequency of the wave.

Differentiating equation (2) with respect to ‘t’ twice

$$\frac{d^2\Psi}{dt^2} = -A\Omega^2 e^{i(kx-\Omega t)} = -\Omega^2\Psi \quad \rightarrow (3)$$

The equation of a travelling wave is

$$\frac{d^2 y}{dx^2} = \frac{1}{\mathrm{v}^2}\frac{d^2 y}{dt^2}$$

Where ‘y’ is the displacement and ‘v’ is the velocity.

Similarly for the de-Broglie wave associated with the particle

$$\frac{d^2\Psi}{dx^2} = \frac{1}{\mathrm{v}^2}\frac{d^2\Psi}{dt^2} \quad \rightarrow (4)$$

where ‘ψ’ is the displacement at time ‘t’.

From eqns (3) & (4)

$$\frac{d^2\Psi}{dx^2} = -\frac{\Omega^2}{\mathrm{v}^2}\Psi$$

But ω = 2πυ and v =υ λ where ‘υ’ is the frequency and ‘λ’ is the wavelength.

$$\frac{d^2\Psi}{dx^2} = -\frac{4\Pi^2}{\Lambda^2}\Psi \text{ or } \frac{1}{\Lambda^2} = -\frac{1}{4\Pi^2\Psi}\frac{d^2\Psi}{dx^2} \quad \rightarrow (5)$$

$$K.E = \frac{1}{2}m\mathrm{v}^2 = \frac{m^2\mathrm{v}^2}{2m} = \frac{P^2}{2m} \quad \rightarrow (6)$$

$$= \frac{h^2}{2m\Lambda^2} \quad \rightarrow (7)$$

Using eqn (5)

$$K.E = \frac{h^2}{2m}\left(-\frac{1}{4\Pi^2\Psi}\frac{d^2\Psi}{dx^2}\right) = -\frac{h^2}{8\Pi^2 m\Psi}\frac{d^2\Psi}{dx^2} \rightarrow (8)$$

Total Energy E = K.E + P.E

$$E = -\frac{h^2}{8\Pi^2 m\Psi}\frac{d^2\Psi}{dx^2} + V$$

$$E - V = -\frac{h^2}{8\Pi^2 m\Psi}\frac{d^2\Psi}{dx^2}$$

$$\frac{d^2\Psi}{dx^2} = -\frac{8\Pi^2 m}{h^2}(E - V)\Psi$$

$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 m}{h^2}(E - V)\Psi = 0}$$

This is the time independent Schrodinger wave equation

11). Using Heisenberg uncertainty principle, prove that electron cannot exist in the nucleus. (Dec2014)

According to the theory of relativity, the energy E of a particle is: $E = mc^2 = \frac{m_o c^2}{\sqrt{1 - (v^2/c^2)}}$

Where 'm_o' is the rest mass of the particle and 'm' is the mass when its velocity is 'v'.

i.e. $$E^2 = \frac{m_o^2 c^4}{1 - (v^2/c^2)} = \frac{m_o^2 c^6}{c^2 - v^2} \rightarrow (1)$$

If 'p' is the momentum of the particle:

i.e. $p = mv = \frac{m_o v}{\sqrt{1 - (v^2/c^2)}}$

$$p^2 = \frac{m_o^2 v^2 c^2}{c^2 - v^2}$$

Multiply by c^2

$$p^2c^2 = \frac{m_o^2 v^2 c^4}{c^2 - v^2} \rightarrow (2)$$

Subtracting (2) by (1) we have

$$E^2 - p^2c^2 = \frac{m_o^2 c^4 (c^2 - v^2)}{c^2 - v^2}$$

$$E^2 = p^2c^2 + m_o^2c^4 \rightarrow (3)$$

Heisenberg's uncertainty principle states that

$$\Delta x \,.\, \Delta P_x \geq \frac{h}{4\Pi} \rightarrow (4)$$

The diameter of the nucleus is of the order 10^{-14}m. If an electron is to exist inside the nucleus, the uncertainty in its position Δx must not exceed 10^{-14}m.

i.e. $\Delta x \leq 10^{-14}$m

The minimum uncertainty in the momentum

$$(\Delta P_x)_{\min} \geq \frac{h}{4\Pi\ (\Delta x)_{\max}} \geq \frac{6.63\times10^{-34}}{4\Pi\times10^{-14}} \geq 0.5 \times 10^{-20}\ \text{kg.m/s} \rightarrow (5)$$

By considering minimum uncertainty in the momentum of the electron

i.e., $(\Delta P_x)_{\min} \geq 0.5 \times 10^{-20}$ kg.m/s = p → (6)

Consider eqn (3)

$$E^2 = p^2c^2 + m_o^2c^4 = c^2(p^2+m_o^2c^2)$$

Where $m_o = 9.11 \times 10^{-31}$ kg

If the electron exists in the nucleus its energy must be

$$E^2 \geq (3 \times 10^8)^2[(0.5 \times 10^{-20})^2 + (9.11 \times 10^{-31})^2(3 \times 10^8)^2]$$

i.e. $E^2 \geq (3 \times 10^8)^2[0.25 \times 10^{-40} + 7.4629 \times 10^{-44}]$

Neglecting the second term as it is smaller by more than the 3 orders of the magnitude compared to first term.

Taking square roots on both sides and simplifying

$$E \geq 1.5 \times 10^{-12}\ \mathrm{J} \geq \frac{1.5\times 10^{-12}}{1.6\times 10^{-19}}\ \mathrm{ev} \geq 9.4\ \mathrm{Mev}$$

If an electron exists in the nucleus its energy must be greater than or equal to 9.4Mev. It is experimentally measured that the beta particles ejected from the nucleus during beta decay have energies of about 3 to 4 MeV. This shows that electrons cannot exist in the nucleus.

12). Calculate the wavelength associated with an electron having K.E 100ev (Dec2014)

$$\Lambda = \frac{h}{\sqrt{2mE}} = \frac{6.6.25\times 10^{-34}}{\boxed{?}\,2\times 9.1\times 10^{-31}\times 100\times 1.6\times 10^{-19}} = 1.23\mathring{A}$$

13). Calculate the wavelength associated with an electron having K.E 100ev (Dec2014)

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14)Write the assumptions of quantum theory of radiation and deduce Ray-Jeans law from Planck's law. (June2015)

Planck assumed that walls of the experimental blackbody consists larger number of electrical oscillators. Each oscillator vibrates with its own frequency.

i) Each oscillator has an energy given by integral multiple of hυ where h is Planck's constant & υ is the frequency of vibration.

$$E = nh\upsilon \text{ where } n = 1, 2, 3 \ldots \text{etc.}$$

ii) An oscillator may lose or gain energy by emitting or absorbing respectively a radiation of frequency υ where $\upsilon=\Delta E/h$, ΔE is difference in energies of the oscillator before and after the emission or absorption take place.

For longer wavelengths $\upsilon = c/\lambda$ is small.
When υ is small $h\upsilon/kT$ is very small.
Expanding $e^{h\upsilon/kT}$ as power series:
$e^{h\upsilon/kT} = 1 + h\upsilon/kT + (h\upsilon/kT)^2 + \ldots$
$\approx 1 + h\upsilon/kT$.
If $h\upsilon/kT$ is small, its higher powers are neglected.

$$e^{h\upsilon/kT}-1 = \frac{hN}{kT} = \frac{hc}{\Lambda kT}$$

Substituting in eqn 1:

$$U_\lambda d\lambda = \left(\frac{8\pi hc}{\lambda^5\left(\frac{hc}{\lambda k}\right)}\right)d\lambda$$

$$= \left(\frac{8\pi kT}{\lambda}\right)d\lambda$$

This is Rayleigh Jeans Law of Radiation

15). Give four important properties of matter waves. (June2015)

1. Waves associated with moving particles are called matter waves. The wavelength ‘λ’ of a de-Broglie wave associated with particle of mass ‘m’ moving with velocity 'v' is

   $\lambda = h/(mv)$

2. Matter waves are not electromagnetic waves because the de Broglie wavelength is independent of charge of the moving particle.
3. The velocity of matter waves (v_P) is not constant. The wavelength is inversely proportional to the velocity of the moving particle.
4. Lighter the particle, longer will be the wavelength of the matter waves, velocity being constant.
5. For a particle at rest the wavelength associated with it becomes infinite. This shows that only moving particle produces the matter waves.

16). Setup time independent Schrödinger wave equation for free particle in one dimension. Consider a particle of mass ‘m’ moving with velocity ‘v’ The de-Broglie wavelength ‘λ’ is**(June2015)**

$$\lambda = \frac{h}{mv} = \frac{h}{P} \rightarrow (1)$$

Where ‘mv’ is the momentum of the particle.

The wave eqn is

$$\Psi = A\,e^{i(kx-\Omega t)} \rightarrow (2)$$

Where ‘A’ is a constant and ‘ω’ is the angular frequency of the wave.

Differentiating equation (2) with respect to ‘t’ twice

$$\frac{d^2\Psi}{dt^2} = -A\Omega^2 e^{i(kx-\Omega t)} = -\Omega^2\Psi \rightarrow (3)$$

The equation of a travelling wave is

$$\frac{d^2y}{dx^2} = \frac{1}{\mathrm{v}^2}\frac{d^2y}{dt^2}$$

Where ‘y’ is the displacement and ‘v’ is the velocity.

Similarly for the de-Broglie wave associated with the particle

$$\frac{d^2\Psi}{dx^2} = \frac{1}{\mathrm{v}^2}\frac{d^2\Psi}{dt^2} \rightarrow (4)$$

where ‘ψ’ is the displacement at time ‘t’.

From eqns (3) & (4)

$$\frac{d^2\Psi}{dx^2} = -\frac{\Omega^2}{\mathrm{v}^2}\Psi$$

But ω = 2πυ and v =υ λ where ‘υ’ is the frequency and ‘λ’ is the wavelength.

$$\frac{d^2\Psi}{dx^2} = -\frac{4\Pi^2}{\Lambda^2}\Psi \text{ or } \frac{1}{\Lambda^2} = -\frac{1}{4\Pi^2\Psi}\frac{d^2\Psi}{dx^2} \rightarrow (5)$$

$$K.E = \frac{1}{2}m\mathrm{v}^2 = \frac{m^2\mathrm{v}^2}{2m} = \frac{P^2}{2m} \rightarrow (6)$$

$$= \frac{h^2}{2m\Lambda^2} \rightarrow (7)$$

Using eqn (5)

$$K.E = \frac{h^2}{2m}\left(-\frac{1}{4\Pi^2\Psi}\right)\frac{d^2\Psi}{dx^2} = -\frac{h^2}{8\Pi^2 m\Psi}\frac{d^2\Psi}{dx^2} \rightarrow (8)$$

Total Energy E = K.E + P.E

$$E = -\frac{h^2}{8\Pi^2 m\Psi}\frac{d^2\Psi}{dx^2} + V$$

$$E - V = -\frac{h^2}{8\Pi^2 m\Psi}\frac{d^2\Psi}{dx^2}$$

$$\frac{d^2\Psi}{dx^2} = -\frac{8\Pi^2 m}{h^2}(E - V)\Psi$$

$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 m}{h^2}(E - V)\Psi = 0}$$

This is the time independent Schrodinger wave equation

17). Calculate the energy in ev , for the first excited state of an electron in an infinite potential well of width 2A^{O} (June2015)

$E_1 = \frac{h^2}{8ma^2}$, $= \frac{6.625X10^{-27}}{8X2X10^{-10}X9.1X10^{-31}}$ $=6.038X10^{-18}$ =37.7eV

18). State de Broglie hypothesis and Show that the group velocity of the de Broglie waves of a partical is equal velocity of the particle. (june2015).

Light exhibits the phenomenon of interference, diffraction, polarization, photoelectric effect and Compton effect.

The phenomenon of interference, diffraction and polarization could be explained only on the basis of wave theory of light. Thus the light exhibit wave nature. Other phenomenon like photoelectric effect, Compton effect etc. Could only be explained through the quantum theory of light. According to which a beam of light consists of small packets each having energy hν. These packets are called photons which have properties like particles. Thus photo electric effect and Compton effect indicate that light possess particle nature. Hence we conclude that light exhibits dual nature.

De Broglie suggested that matter also exhibits dual nature like light radiations at some other instants. In other words particles of matter like electrons and protons also exhibits wave properties. According to de Broglie hypothesis, a moving particle is associated with waves the waves associated with the particles of mater like electrons are known as matter waves or deBroglie waves. This is given by

$$\Lambda = \frac{h}{P} = \frac{h}{mv}$$

The above equation is known as de Broglie wave equation. Hence de Broglie wavelength depends upon the mass of the article and its velocity.

Relation between group velocity and particle velocity:

The equation for group velocity is

$$v_{group} = \frac{d\Omega}{dk} \rightarrow (1)$$

But $\omega = 2\pi\upsilon = 2\pi(E/h) \rightarrow (2)$

$$d\Omega = \frac{2\Pi}{h} dE \rightarrow (3)$$

We have $k = 2\pi/\lambda = 2\pi(p/h) \rightarrow (4)$

$$dk = \frac{2\Pi}{h} dP \rightarrow (5)$$

Dividing eqn (3) by (5) we have

$$\frac{d\Omega}{dk} = \frac{dE}{dP} \rightarrow (6)$$

But we have $E = P^2/(2m)$, Where 'P' is the momentum of the particle.

$$\frac{dE}{dP} = \frac{2P}{2m} = \frac{P}{m}$$

Using the above in eqn (6)

$$\frac{d\Omega}{dk} = \frac{P}{m}$$

But $p = mv_{particle}$, Where '$v_{particle}$' is the velocity of the particle.

$$\frac{d\Omega}{dk} = \frac{mv_{particle}}{m} = v_{particle} \rightarrow (7)$$

From eqn (1) & (7), we have

$$v_{group} = v_{particle} \rightarrow (8)$$

∴ The de Broglie's wave group associated with a particle travels with a velocity equal to the velocity of the particle itself.

19). State and explain Heisenberg uncertainity principle . (June2015).

According to classical mechanics a particle occupies a definite place in space and possesses a definite momentum. If the position and momentum of a particle is known at any instant of time, it is possible to calculate its position and momentum at any later instant of time. The path of the particle could be traced. This concept breaks down in quantum mechanics leading to Heisenberg's Uncertainty Principle according to which "It is impossible to measure simultaneously both the position and momentum of a particle accurately. If we make an effort to measure very accurately the position of a particle, it leads to large uncertainty in the measurement of momentum and vice versa.

If Δx and ΔP_x are the uncertainties in the measurement of position and momentum of the particle then the uncertainty can be written as

$$\Delta x . \Delta P_x \geq (h/4\pi)$$

In any simultaneous determination of the position and momentum of the particle, the product of the corresponding uncertainties inherently present in the measurement is equal to or greater than $h/4\pi$.

Similarly, 1) $\Delta E.\Delta t \geq h/4\pi$ This equation represents uncertainty in energy and time. ΔE is uncertainty in energy,Δt is the uncertainty in time.

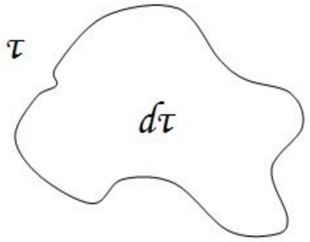
2) $\Delta L.\Delta\theta \geq h/4\pi$ This equation represents uncertainty in angular momentum(ΔL) and angular displacement($\Delta\theta$)

20). Explain in brief the properties of wave function . If the wave function of a partical in an infinite potential box of width a is Ψ=Bsin(nΠX/a) where X is the position aand n is the quantum number , find B.

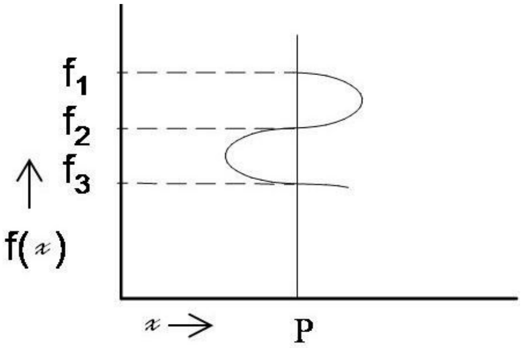
(June2015)

Properties of the wave function:

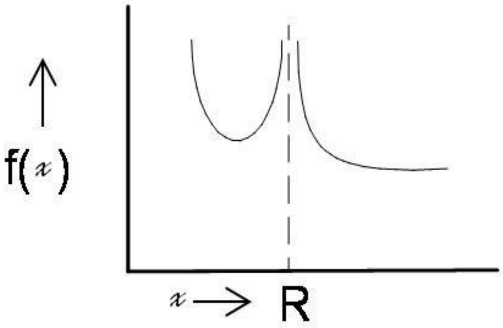
A system or state of the particle is defined by its energy, momentum, position etc. If the wave function 'ψ' of the system is known, the system can be defined. The wave function 'ψ' of the system changes with its state. To find 'ψ' Schrodinger equation has to be solved. As it is a second order differential equation, there are several solutions. All the solutions may not be correct. We have to select those wave functions which are suitable to the system. The acceptable wave function has to possess the following properties:



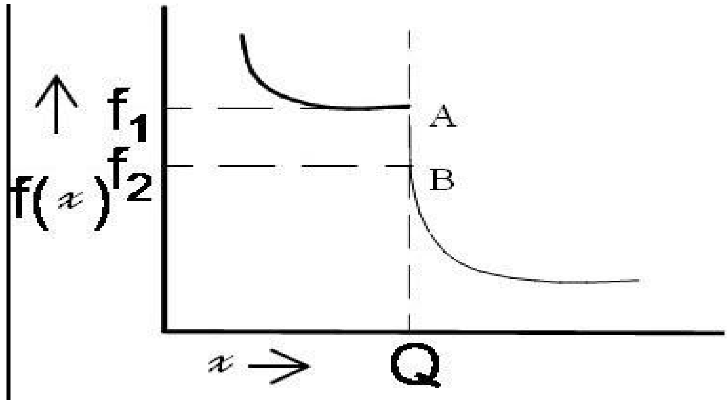
1)'ψ' is single valued everywhere: Consider the function f(x) which varies with position as represented in the graph. The function f(x) has three values f_1, f_2 and f_3 at x = p. Since $f_1 \neq f_2 \neq f_3$ it is to state that if f(x) were to be the wave function. The probability of finding the particle has three different values at the same location which is not true. Thus the wave function is not acceptable.



2)'ψ' is finite everywhere: Consider the function f(x) which varies with position as represented in the graph. The function f(x) is not finite at x=R but f(x)=∞. Thus it indicates large probability of finding the particle at a location. It violates uncertainty principle. Thus the wave function is not acceptable.

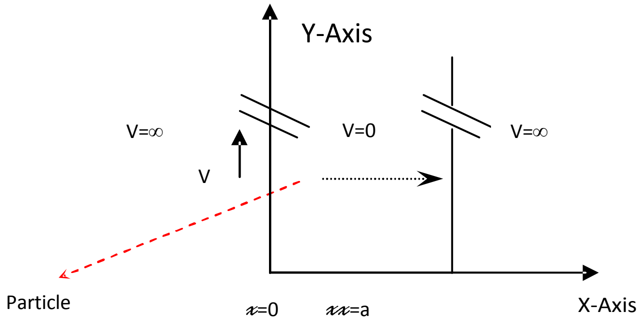


3)'ψ' and its first derivatives with respect to its variables are continuous everywhere: Consider the function f(x) which varies with position as represented in the graph. The function f(x) is truncated at x=Q between the points A & B, the state of the system is not defined. To obtain the wave function associated with the system, we have to solve Schrodinger wave equation. Since it is a second order differential wave equation, the wave function and its first derivative must be continuous at x=Q. As it is a discontinuous wave function, the wave function is not acceptable.



For bound states ‘ψ’ must vanish at potential boundary and outsde. If ‘ψ*’ is a complex function, then ψ* ψ must also vanish at potential boundary and outside.

Energy Eigen values of a particle in one dimensional, infinite potential well (potential well of infinite depth) or of a particle in a box.



Consider a particle of a mass ‘m’ free to move in one dimension along positive x-direction between x=0 to x=a. The potential energy outside this region is infinite and within the region is zero. The particle is in bound state. Such a configuration of potential in space is called infinite potential well. It is also called particle in a box. The Schrödinger equation outside the well is

$$\frac{d^2\Psi}{dx^2} \underset{\mp}{m} \frac{8\Pi^2}{} (E - \infty)\Psi = 0 \rightarrow (1) \qquad \because \mathrm{V} = \infty$$

For outside, the equation holds good if ψ = 0 & $|\psi|^2 = 0$. That is particle cannot be found outside the well and also at the walls

The Schrodinger's equation inside the well is:

$$\frac{d^2\Psi}{dx^2} \mp \frac{8\Pi^2}{m} E\Psi = 0 \rightarrow (2) \qquad \because V = 0$$

$$-\frac{h^2}{8\Pi^2 m}\frac{d^2\Psi}{dx^2} = E\Psi \rightarrow (3)$$

This is in the form $\hat{H}\psi = E\psi$

This is an Eigen-value equation.

Let $\frac{8\Pi^2}{m} E = k^2$ in eqn (2)

$$\frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

The solution of this equation is:

$\psi = C \cos kx + D \sin kx \rightarrow (4)$

at $x = 0 \rightarrow \psi = 0$

$0 = C \cos 0 + D \sin 0$

$\therefore C = 0$

Also $x = a \rightarrow \psi = 0$

$0 = C \cos ka + D \sin ka$

But C = 0

$\therefore D \sin ka = 0$ (5)

$D \neq 0$ (because the wave concept vanishes) i.e. ka = $n\pi$ where n = 0, 1, 2, 3, 4… (Quantum number)

$$k = \frac{n\Pi}{a} \rightarrow (6)$$

Using this in eqn (4)

$$\Psi_n = D\sin\frac{n\Pi}{a}x \rightarrow (7)$$

Which gives permitted wave functions.

To find out the value of D, normalization of the wave function is to be done.

$$\text{i.e. } \int_0^a |\psi_n|^2 dx = 1 \rightarrow (8)$$

using the values of ψ_n from eqn (7)

$$\int_0^a D^2 \sin^2\frac{n\Pi}{a}x\,dx = 1$$

$$D^2\int_0^a \left[\frac{1-\cos(2n\Pi/a)x}{2}\right]dx = 1$$

$$\sin^2\Theta = \left[\frac{1-\cos 2\Theta}{2}\right]$$

$$\frac{D^2}{2}\left[\int_0^a dx - \int_0^a \cos\frac{2n\Pi}{a}x\,dx\right] = 1$$

$$\frac{D^2}{2}\left[x - \frac{a}{2n\Pi}\sin\frac{2n\Pi}{a}x\right]_0^a = 1$$

$$\frac{D^2}{2}[a-0] = 1$$

$$\frac{D^2}{2}a = 1$$

$$D = \sqrt{\frac{2}{a}}$$

Hence the normalized wave functions of a particle in one dimensional infinite potential well is:

$$\Psi_n = \sqrt{\frac{2}{a}}\sin\frac{n\Pi}{a}x \rightarrow (9)$$

21).The wavelength of a fast neutron of mass 1.675X10-27 kg is 0.02nm. Calculate the group velocity and phase velocity of its de Broglie waves (June2015)

$$V_g = \frac{\boxed{?}}{m_n\lambda} = \frac{6.625X10^{-27}}{1.675X10^{-27}X0.02X10^{-10}} = 1.98X10^{4}\ ms^{-1}$$

$$\mathbf{V_P} = \frac{C^2}{V_G} = \frac{3X10^{8}}{1.98X10^{4}} = 4.55X10^{12}ms^{-1}$$

Module-II
Electrical properties of materials

1).Explainfailures of classical free electron theory.(jan 2013) (July2014)

Electrical and thermal conductivities can be explained from classical free electron theory. It fails to account the facts such as specific heat, temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.

(a)Specific heat capacity

The molar specific heat of metal at constant volume is

$$C_v = \frac{3}{2}R$$

But experimentally the contribution to the specific heat capacity by the conduction electrons is found to be

$$C_v = 10^{-4}\ RT$$

Thus the predicted value is much higher than the experimental value. Further the theory indicates no relationship between the specific heat capacity and temperature. Experimentally however, the specific heat capacity is directly proportional to the absolute temperature.

(b) Dependence of electrical conductivity on temperature:

According to the main assumptions of the classical free electron theory.

$$\frac{3}{2}kT = \frac{1}{2}mv_{th}^{2}$$

$$\square v_{th} = \boxed{?}\frac{3kT}{m}$$

$$v_{th} \square \sqrt{T}$$

The mean collision time T is inversely proportional to the thermal velocity.

$$T \square \frac{1}{vth}$$

$$\square T \square \frac{1}{\sqrt{T}}$$

Electrically conductivity Σ is given by

$$\Sigma = \frac{ne^2\tau}{m}$$

$$\square \Sigma \square T \square \frac{1}{\sqrt{T}}$$

$$\boxed{\Sigma_{\text{theory}} \square \frac{1}{\sqrt{T}}}$$ → Theoretically obtained

But experimentally it is found that $$\boxed{\Sigma_{\text{exp}} \square \frac{1}{T}}$$

Again it is found that the prediction of the theory is not matching with the experimental results.

(c) Dependence of electrical conductivity on electron concentration:

According to classical theory electrical conductivity Σ is given by.

$$\Sigma = \frac{ne^2\tau}{m}$$

Therefore $\sigma \propto n$

This means that divalent and trivalent metals, with larger concentration of electrons should possess much higher electrical conductivity than monovalent metals, which is contradiction to the observed fact. Where experimentally it is found that as 'n' increases, Σ is not found to increase. The conductivity of monovalent metals [E.g. copper, silver] is greater than that of divalent and trivalent metals [E.g.: zinc, aluminium].

****[*The electron concentrations for zinc and cadmium are* $13.1\times10^{28}/m^3$ *and* $9.28\times10^{28}/m^3$ *which are much higher than that for copper and silver, the values of which are* $8.45\times10^{28}/m^3$ *and* $5.85\times10^{28}/m^3$ *respectively. Zinc and cadmium which are divalent metals have conductivities* $1.09\times10^7/\Omega m$ *and* $0.15\text{x}10^7/\Omega m$*. These are much lesser than that of monovalent metals copper and silver the values of which are* $5.88\text{x}10^7/\Omega m$ *and* $6.3\text{x}10^7/\Omega m$ *respectively.*]

2). What are the merits of quantum free electron theory?(Jan 2013)

a) Specific heat:

According to classical free electron theory all the free electrons in a metal absorb the heat energy when a metal is heated. It results in a large value of specific heat. But as per quantum free electron theory, only a few electrons that are occupying energy levels close to Fermi energy level

E_F absorb the heat energy to get excited to higher energy levels and contribute for specific heat. Hence the value of specific heat is very small.

According to quantum free electron theory the specific heat of solids is given by

$$C_v = 10^{-4}RT$$

The above result agrees well with the experimentally observed values.

b) Temperature dependence of electrical conductivity:

We have, $\Sigma = \frac{ne^2\tau}{m\square}$

$\Sigma = \frac{ne^2}{m\square\ v_f}$ .... (1) Because $\tau = \frac{\Lambda}{V_f}$

As per quantum free electron theory, E_f and V_f are independent of temperature. But Λ is dependent on temperature and is explained as.

As the free electrons traverse in a metal they get scattered by vibrating ions of the lattice. The vibrations occur in such a way that the displacement of ions takes place equally in all directions. Hence ions may be assumed to be present effectively in a circular cross section of area Πr^2 which blocks the path of the electrons irrespective of direction of approach [here r is the amplitude of vibration].

The vibrations of larger area of cross section scatter more effectively, thereby reducing Λ

$$\square\Lambda\square\frac{1}{\Pi r}\text{………. (2)}$$

But

a) The energy E of the vibrating body is proportional to the square of amplitude

b) E is due to thermal energy.

c) E is proportional to temperature T.

$$\square r^2 \square\ T$$

$$\square\Lambda\square\frac{1}{T}\text{………..…..(3)}$$

Comparing (1) & (3)

Thus $\Sigma\square\frac{1}{T}$

Thus the exact dependence of Σ on T is explained.

c). Electrical conductivity and electron concentration:

The free electron model suggests that Σ is proportional to electron concentration 'n' but trivalent metals such as aluminum and gallium having more electron concentration than that of monovalent metals, have lower electrical conductivity than monovalent metals such as copper and silver. According to quantum free electron theory.

$$\Sigma = \frac{ne^{2}}{\,} \square_{\mathrm{v}}\text{-}\square_{f}$$

It is clear from the above equation $^{\Lambda}$that Σ depends on both 'n' & the ratio $\square_{\mathrm{v}}\text{-}\overset{\Lambda}{\square}_{f}$.

3). Define Fermi energyand Fermi factor. Discuss variation of Fermi factor With temperature. (July2013) (July2014)

Under thermal equilibrium the free electrons acquire energy obeying a statistical rule known as Fermi-Dirac statistics.Fermi-Dirac distribution deals with the distribution of electrons among the permitted energy levels. The permitted energy levels are the characteristics of the given material. The density of the state function g(E) changes with energy in a band. The number of energy levels in the unit volume of the material in the energy range E & (E+dE) is g(E)dE.

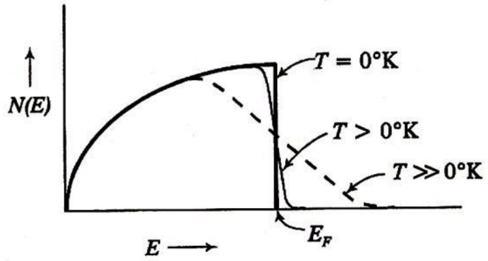
Each electron will have its own energy value which is different from all others except the one with opposite spin. The number of electrons with energy range E &(E+dE) in unit volume is N (E) dE which is the product of the number of energy levels in the same range and the fermi factor.

$\therefore$ N (E) dE = f(E)×g(E)dE.

But f(E) and g(E) at a temperature T changes only with E. That is, N(E)dE at a given temperature change with E.

The plot of N(E)dE versus E represents the actual distribution of electrons among the available states for the material at different temperatures. The distribution is known as Fermi–Dirac distribution. Fermi-Dirac distribution represents the detailed distribution of electrons among the various available energy levels of a material under thermal equilibrium conditions.

Fermi-Dirac distribution can be considered in the following three conditions: At T=0K, at T> 0K and T>>0K.The plot of N(E) versus E for all the three cases is in the fig.



FERMI-DIRAC DISTRIBUTION AT DIFFERENT TEMPERATURES

Fermi factor,f (E)**:**The electrons in the energy levels far below Fermi level cannot absorb the energy above absolute zero temperature. At ordinary temperature because there are no vacant energy levels above Fermi level into which electrons could get into after absorbing the thermal energy. Though the excitations are random, the distributions of electrons in various energy levels will be systematically governed by a statistical function at the steady state.

The probability f(E) that a given energy state with energy E is occupied at a steady temperature T is given by, $f(E)=\frac{1}{e^{\frac{(E-E_F)}{kT}}+1}$ f(E) is called the Fermi factor.

Thus, "*Fermi factor is the probability of occupation of a given energy state for a material in thermal equilibrium*".

(i) Probability of occupation for $E<E_f$ at T = 0°K

When T=0°K &$E<E_f$

$$f(E) = \frac{1}{e^{-\infty}+1}$$

$$= \frac{1}{0+1}$$

f(E) = 1 for $E < E_F$

Hence,f(E) = 1 means the energy level is certainly occupied, & $E<E_F$ applies to all the levels below E_F.

□ At T=0°K, all the energy levels *below* the Fermi level are *all occupied*.

(ii) Probability of occupation for $E > E_F$ at T= 0°K

When T =0°K & $E > E_F$

$$f(E) = \frac{1}{e^{\infty}+1} = \frac{1}{\infty}$$

$f(E) = 0$ for $E > E_F$

□ At T=0°K all the energy levels *above* Fermi level are *unoccupied*.

In view of the above two cases, at T=0°K the variation of f(E) for different energy values, becomes a step function as in fig.

(iii) Probability of occupation at ordinary temperatures.

At ordinary temperatures, though the value of probability remains 1 for $E \ll E_F$ it starts decreasing from 1 as the values of E become closer to E_F.

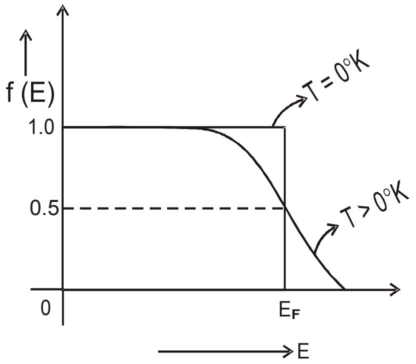
At $E = E_F$ & $T > 0°$ k

$$f(E) = \frac{1}{e^{\frac{0}{kT}} + 1}$$

$$= \frac{1}{1+1}$$

$$f(E) = \frac{1}{2}$$

Further, for $E > E_f$, the probability value falls off to zero rapidly. Thus plot of f(E) Vs E at different temperatures.



Fermi Temperature:

*"Fermi temperature is the temperature at which the average thermal energy of the free electron in a solid becomes equal to the Fermi energy at 0°K ".*But the thermal energy possessed by electrons is given by the product of kT.

It means when $T = T_F$, $kT_F = E_{F0}$ is satisfies.

But all practical purposes, $\mathbf{E_F = E_{Fo}}$

□ $\mathbf{T_F} = \frac{E_F}{k}$

Fermi temperature is only a hypothetical concept because even at ordinary temperature it is not possible for electrons to receive thermal energy in a magnitude of EF. For example at EF = 3ev TF = 34800 K which is quite exaggerated to realize.

18). What is mean collision time? Using free electron theory in a metal, obtain an expression electrical conductivity in terms of mean collision time.(July2013)

"The average time that elapses between two consecutive collisions of an electron with the lattice points is mean collision time." It is given as

$\mathrm{T} = \frac{\Lambda}{v}$ Where Λ - mean free path

v -total velocity of the electrons due to the combined effect of thermal and drift velocities.

If a constant electric field E is applied to the metal the electron of mass m and charge e will experience a force, F= - eE.

F= - eE driving force on the electron which results in the drift velocity v_d,

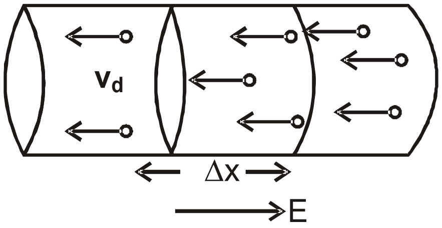
Then the resistance force offered to its motion is

$F_r = \frac{mv_d}{\tau}$ where τ mean collision time

In the steady state,

$F_r = F$

i,e, $\frac{mv_d}{\tau} = eE$



$$\boxed{V_d = \left(\frac{eE}{m}\right)\tau}$$

Note : Current density, $J = \frac{I}{A}$

$$\boxed{J = nev_d} \qquad \boxed{I = nAev_d}$$

Electrical conductivity in metals

$$\boxed{\sigma = \frac{ne^2\tau}{m}}$$

4).What are the assumptions made in quantum free electron theory? Explain success this theory. (Dec 2014)

Assumptions:

1. The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.
2. The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle and also obey the Fermi – Dirac quantum statistics.
3. The field due to positive ion core is constant throughout the material. So electrons travel in a constant potential inside the metal but stay confined within its boundaries.

4. Both the attraction between the electrons and the lattice points, the repulsion between the electrons themselves is ignored and therefore electrons are treated free.

Merits of Quantum Free Electron Theory:

a. **Specific heat:**

According to classical free electron theory all the free electrons in a metal absorb the heat energy when a metal is heated. It results in a large value of specific heat. But as per quantum free electron theory, only a few electrons that are occupying energy levels close to Fermi energy level E_F absorb the heat energy to get excited to higher energy levels and contribute for specific heat. Hence the value of specific heat is very small.

According to quantum free electron theory the specific heat of solids is given by

$$C_v = 10^{-4}RT$$

The above result agrees well with the experimentally observed values.

b.**Temperature dependence of electrical conductivity**:

We have, $\Sigma = \frac{ne^2\tau}{m\square}$

$$\Sigma = \frac{ne^2}{m\square\, v_f} \text{ ………… (1)} \qquad \text{Because } \tau = \frac{\Lambda}{v_f}$$

As per quantum free electron theory, E_f and V_f are independent of temperature. But Λ is dependent on temperature and is explained as.

As the free electrons traverse in a metal they get scattered by vibrating ions of the lattice. The vibrations occur in such a way that the displacement of ions takes place equally in all directions. Hence ions may be assumed to be present effectively in a circular cross section of area Πr^2 which blocks the path of the electrons irrespective of direction of approach [here r is the amplitude of vibration].

The vibrations of larger area of cross section scatter more effectively, thereby reducing Λ

$$\square\Lambda\square\frac{1}{\Pi r} \text{ ………. (2)}$$

But

d) The energy E of the vibrating body is proportional to the square of amplitude
e) E is due to thermal energy.
f) E is proportional to temperature T.

$$\square r^2\square\ T$$

$$\square\Lambda\square\frac{1}{T} \text{ ………..…..(3)}$$

Comparing (1) & (3)

Thus $\Sigma\square\frac{1}{T}$

Thus the exact dependence of Σ on T is explained.

c. **Electrical conductivity and electron concentration**:

The free electron model suggests that Σ is proportional to electron concentration ‘n’ but trivalent metals such as aluminum and gallium having more electron concentration than that of monovalent metals, have lower electrical conductivity than monovalent metals such as copper and silver. According to quantum free electron theory.

$$\Sigma = \frac{ne^2}{\;} \boxed{?}_{v}\text{-}\boxed{?}_{f}$$

It is clear from the above equation that Σ depends on both ‘n’ & the ratio $\boxed{?}_{v}\text{-}\hat{\boxed{?}}_{f}$.

5). What is Fermi level? Describe the variation of Fermi factor with temperatures(Dec 2014)

“*The energy of the highest occupied level at absolute zero temperature (0K) is called the Fermi energy and the energy level is called Fermi level”. It is denoted by 'E_f'*

*Fermi factor f (E)***:** The electrons in the energy levels far below Fermi level cannot absorb the energy above absolute zero temperature. At ordinary temperature because there are no vacant energy levels above Fermi level into which electrons could get into after absorbing the thermal energy. Though the excitations are random, the distributions of electrons in various energy levels will be systematically governed by a statistical function at the steady state.

The probability f(E) that a given energy state with energy E is occupied at a steady temperature T is given by, $f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$ f(E) is called the fermi factor.

Thus, “*Fermi factor is the probability of occupation of a given energy state for a material in thermal equilibrium”.*

(iv)Probability of occupation for $E<E_f$ at T = 0°K

When T=0°K &$E<E_f$

$$f(E) = \frac{1}{e^{-\infty}+1}$$

$$= \frac{1}{0+1}$$

$f(E) = 1$ for $E < E_F$

Hence,f(E) = 1 means the energy level is certainly occupied, & $E<E_F$ applies to all the levels below E_F.

- At T=0°K, all the energy levels *below* the Fermi level are *all occupied*.

(v) Probability of occupation for $E > E_F$ at T= 0°K

When T =0°K & $E > E_F$

$$f(E) = \frac{1}{e^{\infty}+1} = \frac{1}{\infty}$$

$f(E) = 0$ for $E > E_F$

- At T=0°K all the energy levels *above* Fermi level are *unoccupied*.

In view of the above two cases, at T=0°K the variation of f(E) for different energy values, becomes a step function as in fig.

(vi) Probability of occupation at ordinary temperatures.

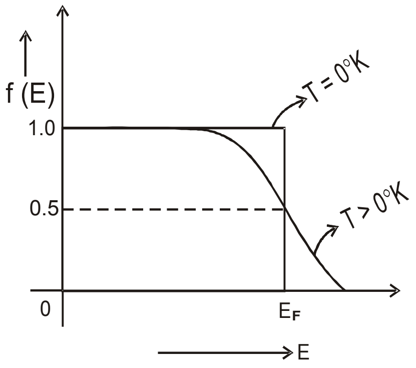
At ordinary temperatures, though the value of probability remains 1 for $E \ll E_F$ it starts decreasing from 1 as the values of E become closer to E_F.

At $E = E_F$ & $T > 0°$ k

$$f(E) = \frac{1}{e^{\frac{0}{kT}} + 1}$$

$$= \frac{1}{1+1}$$

$$f(E) = \frac{1}{2}$$



Further, for $E > E_f$, the probability value falls off to zero rapidly. Thus plot of f(E) Vs E at different temperatures.

6). Explain the Meissner effect and different types of superconductors. (Dec 2014)

Meissner effect:

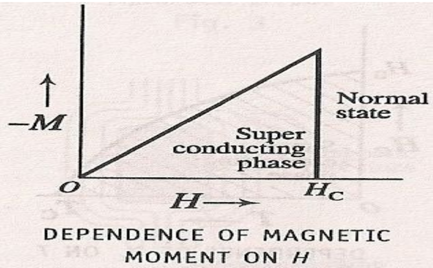
A superconducting material kept in a magnetic field expels the magnetic flux out of its body when it is cooled below the critical temperature and thus becomes perfect diamagnet. This effect is called Meissner effect.

Type of superconductors

1.Type I superconductors:

These materials exhibit complete Meissner effect and have well defined critical field H_c. These are perfect diamagnetic in the superconducting state and possesses negative magnetic moment.

The material remains in the superconducting when the field is less than the critical field. It expels the magnetic lines of force from the body of the mater immediately after H_c the material transits to normal state and the flux penetrates the material i.e. Meissner effect is absent. H_c is of the order of 0.1 T or less. Since H_c very low, even weak magnetic field can destroy the phenomenon. As weak magnetic field can penetrate the material more easily,they are also called *soft superconductor.*

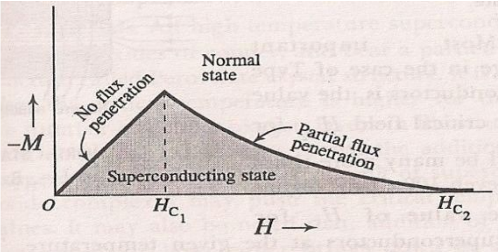


DEPENDENCE OF MAGNETIC MOMENT ON H

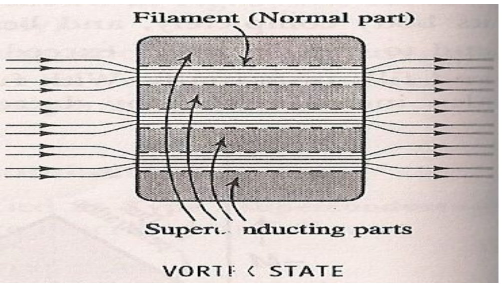
2.Type II Superconductors:

These materials are having two critical fields H_{c1} and H_{c2}. For the field less then H_{c1}(lower critical field), it expels the magnetic field completely and there is no flux penetration. It becomes a perfect diamagnetic and the material is in the super conducting state. After Hc_1 the flux penetrates and partially fills the body of the material through channels called filaments. As the field is increased these filaments broaden and by Hc_2 (the material possesses both normal and superconducting state, hence the state is called *'Mixed State'*. This is also referred to as '*vortex state'* where the material is in a magnetically mixed state but electrically it is a superconductor. After Hc_2 the material transits to normal state and the resistance is finite.Type II superconductors can carry larger currents when the magnetic field is between Hc_1 and Hc_2.

Hc_2 the upper critical field is many a folds greater than Hc_1 the lower critical field. Only strong magnetic field of the order of 10T can penetrate the material hence these are called hard super conductor.Type-II superconductors are used in the manufacturing of the superconducting magnets of high magnetic fields above 10 Tesla.



DEPENDENCE OF MAGNETIC MOMENT ON H



VORTEX STATE

Element	T_c (K)
Tantalum	4.5
Thallium	2.4
Niobium	9.3

7) The electron concentration in an n-type semiconductor is $5X10^{17}/m^3$.Neglecting the hole current; calculate the conductivity of the material if the drift velocity of the electrons is 350 m/s in an electric field of 100 V/m. (Dec 2014)

$\Sigma = N_e\, e\, \mu_e = N_e\, e[\frac{V}{E}] = 5x10^{17}x1.6x10^{-19}x\frac{100}{350} = 0.028/\Omega m$

8) What is Superconductivity? Explain the superconductivity on the basis of BCS theory. (Dec 2014)

Super conductivity is the phenomenon observed in some metals and materials. KammerlinghOnnes in 1911 observed that the electrical resistivity of pure mercury drops abruptly to zero at about 4.2K .This state is called super conducting state. The material is called superconductor .The temperature at which they attain superconductivity is called critical temperature T_c.

BCS Theory (Explanation of superconductivity)*:*

Bardeen, Cooper and Schrieffer (BCS) in 1957 explained the phenomenon of superconductivity based on the formation of cooper pairs. It is called BCS theory. It is a quantum mechanical concept.

During the flow of current in a superconductor, when an electron approaches a positive ion lattice of the metal there is a Coulomb force of attraction between the electron and the lattice ion and thus ion core is set in motion causing lattice distortion. Smaller the mass of the positive core, larger will be the distortion. The lattice vibrations are quantized in a term of Phonons. Now another electron passing by this distorted lattice will interact with it and thus the energy of this electron is also reduced. This interaction is looked upon as if the two electrons interact via the phonon field, (because the lattice vibrations are quantized and quanta of these vibrations are phonons) resulting in lowering of energy for the electrons. Due to this interaction an apparent force of attraction develops between the electrons and they tend to move in pairs.This interaction is strongest when the two electronic have equal and opposite spins and momenta.This leads to the formation of cooper pairs. “Cooper pairs are a bound pair of electrons formed by the interaction between the electrons with opposite spin and momenta in a phonon field”.

At normal temperatures the attractive force is too small and pairing of electrons does not take place.At lower temperature that is below the critical temperature the apparent force of attraction exceeds the Colombian force of repulsion between two electrons leading to the formation of cooper pairs. According to quantum mechanics a cooper pair is treated as single entity. A wave function is associated with each such cooper pair and wave functions associated with similar cooper pairs start overlapping which extends over a million pairs and hence virtually over the entire volume of the superconductor. Finally large number of cooper pairs form a union one aiding the motion of the other. So the entire union of cooper pairs will therefore move as one unit. The resistance experienced by any one cooper pair is overcome by the co-operative action of the other pairs in the union. Ultimately when electrons flow in a material in the form of cooper pairs do not encounter scattering. The resistance vanishes and conductivity is very large and thus the phenomenon superconductivity.

9). Explain the laws of mass action and derive the conductivity expression of a semiconductor. (Dec 2014)

"The law says that, for a given semiconductor material either extrinsic or intrinsic, the product of the charge carrier concentration remains a constant at any temperature, even if the doping is varied".

That is, if N_e& N_h are the electron and hole concentrations respectively,

Then $N_e N_h$ **= a constant.**

This condition is applicable for both extrinsic and intrinsic semiconductors.

For an intrinsic semiconductor, $N_e = N_h = n_i$ where, n_i is the intrinsic charge carrier density.

Then $\mathbf{N_e N_h = n^2_i}$ **= a constant.**

Expression for Electrical Conductivity of a Semiconductor:

Let us consider to start with, the conductivity in a semiconductor due to the flow of electrons only. Consider a semiconductor of area of cross section A, in which a current I is flowing. Let v be the velocity of electrons. The electrons move through a distance v in one second. As per the assumption of free electron theory, a large number of free electrons flow freely through the semiconductor whose area of cross section is A.

The volume swept by the electrons/second = Av.

If N_e is the number of electrons / unit volume, and e is the magnitude of electric charge on the electron, then, the charge flow/second = N_eAv.

Since charge flow/second is the current I,

$I = N_e eAv$.

□ Current density $J = (I/A) = N_e ev$ …… (1)

The electron mobility, μe is given by,

$\mu_e = v/E$, …… (2)

where, E is the electric field.

Substituting for v, from Eq. (2), Eq. (1) becomes,

$J = N_e e (\mu_e E)$ …… (3)

But the ohm's law, is given by the equation,

$J = \Sigma E$ where Σ is the conductivity of the charge carriers.

If Σ_e is the conductivity due to electrons in the semiconductor material, then ohm's law becomes,

$J = \Sigma_e E$, …… (4)

Comparing Eqs (3) and (4),
conductivity of electrons is given by,

$\Sigma_e = N_e e\mu_e$ …… (5)

Now let us consider the contribution of holes to the conduction of electricity. If Σ_h is the conductivity due holes, N_h is the number of holes/unit volume and μ_h is the mobility of holes, then in similarity to Eq (5), the equation for conductivity due to holes can be written as,

$\Sigma_h = N_h e \mu_h$ …… (6)

The total conductivity for a semiconductor is given by the sum of Σ_e and Σ_h .

i.e, $\Sigma = \Sigma_e + \Sigma_h = N_e e \mu_e + N_h e \mu_h$

$\boldsymbol{\Sigma = e (N_e \mu_e + N_h \mu_h)}$ …… (7)

10) What is Fermi- Dirac statistics? Explain. (Dec 2014)

Fermi-Dirac statistics: Under thermal equilibrium the free electrons acquire energy obeying a statistical rule known as Fermi-Dirac statistics.Fermi-Dirac distribution deals with the distribution of electrons among the permitted energy levels. The permitted energy levels are the characteristics of the given material. The density of the state function g(E) changes with energy in a band. The number of energy levels in the unit volume of the material in the energy range E & (E+dE) is g(E)dE.

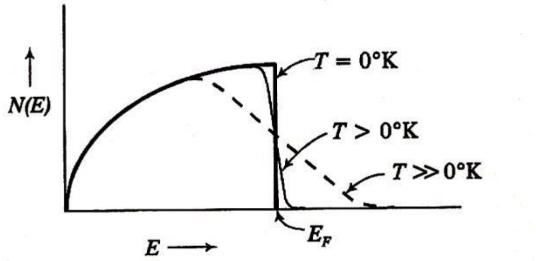
Each electron will have its own energy value which is different from all others except the one with opposite spin. The number of electrons with energy range E & (E+dE) in unit volume is N (E) dE which is the product of the number of energy levels in the same range and the fermi factor.

$\therefore N(E)\,dE = f(E)\times g(E)dE.$

But f(E) and g(E) at a temperature T changes only with E. That is, N(E)dE at a given temperature change with E.

The plot of N(E)dE versus E represents the actual distribution of electrons among the available states for the material at different temperatures. The distribution is known as Fermi–Dirac distribution. Fermi-Dirac distribution represents the detailed distribution of electrons among the various available energy levels of a material under thermal equilibrium conditions.

Fermi-Dirac distribution can be considered in the following three conditions: At T=0K, at T> 0K and T>>0K.The plot of N(E) versus E for all the three cases is in the fig.



FERMI-DIRAC DISTRIBUTION AT DIFFERENT TEMPERATURES

11) The Fermi level in silver is 5.5ev. Find the velocity of electrons in silver. (Dec 2014)

Given: E_F = 5.5ev

$$V_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2X5.5X1.6X10^{-19}}{9.11X10^{-31}}} = 1.38X10^6 \text{ m/s}$$

12) Obtain an expression for the conductivity of a metal from quantum mechanical considerations. (June2015)

If a constant electric field E is applied to the metal the electron of mass m and charge e will experience a force, F= - eE.

F= - eE driving force on the electron which results in the drift velocity v_d,

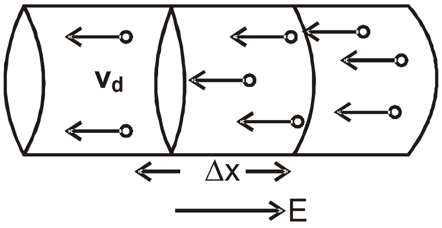
Then the resistance force offered to its motion is

$F_r = \frac{mv_d}{T}$ where T mean collision time

In the steady state,

$F_r = F$

i,e, $\frac{mv_d}{T} = eE$



$$\boxed{V_d = \left(\frac{eE}{m}\right) T}$$

Note : Current density, $J = \frac{I}{A}$

$$\boxed{J = nev_d} \qquad \boxed{I = nAev_d}$$

Electrical conductivity in metals

$$\boxed{\sigma = \frac{ne^2\tau}{m}}$$

Thus conductivity is proportional to the number of electrons per m^3.

13)Show that the Fermi level of an intrinsic semiconductor lies in the middle of the forbidden energy gap. (June2015)

For an intrinsic semiconductor, $N_e = N_h = n_i$

i.e,

$$\frac{4\sqrt{2}}{h^3}(\pi m_e^* kT)^{3/2} e^{\left(\frac{E_F-E_g}{kT}\right)} = \frac{4\sqrt{2}}{h^3}(\pi m_h^* kT)^{3/2} e^{-E_F/kT}$$

$$(m_e^*)^{3/2} e^{\left(\frac{E_F-E_g}{kT}\right)} = (m_h^*)^{3/2} e^{-E_F/kT}$$

or,

$$e^{\left(\frac{2E_F-E_g}{kT}\right)} = (m_h^*/m_e^*)^{3/2}$$

Taking natural logarithm on both sides, we get,

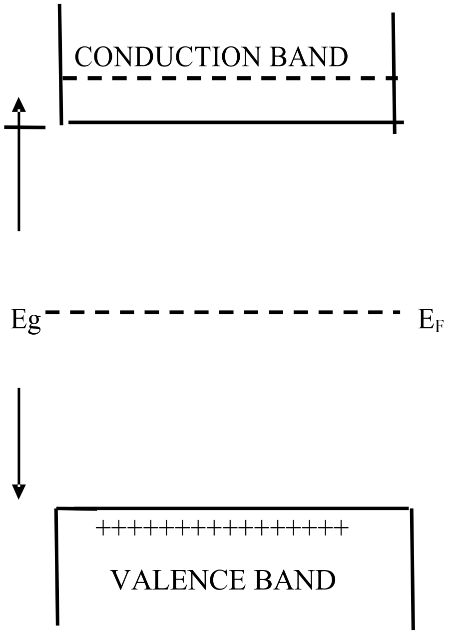
$$\left(\frac{2E_F-E_g}{kT}\right) = \frac{3}{2}\ln\left(\frac{m_h^*}{m_e^*}\right)$$

Or, $\mathbf{E_F} = \frac{3}{4}kT\ln\left(\frac{m_h^*}{m_e^*}\right) + \frac{E_g}{2}$ because $, E_g = E_c + E_v$

Under practical considerations, $m_e^* = m_h^*$, because of which the first term in the right hand side of the above equation goes to zero, and since ln 1=0

$$\mathbf{E_F} = \frac{E_g}{2}$$

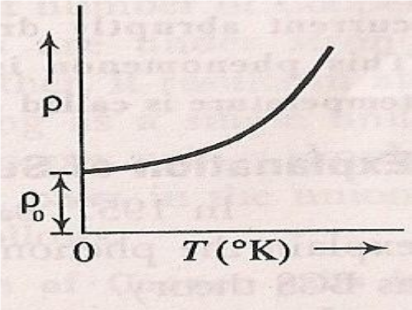
The average energy of the electrons taking part in conduction will be almost equal to (1/2) Eg. Thus the Fermi level lies in the mid-part of the forbidden gap for an intrinsic semiconductor.



FERMI LEVEL IN INTRINSIC SEMICONDUCTOR

14)Explain the temperature dependence of resisitivity of metal and state Matthiessen´s rule. (June2015)

All metals are good conductors of electricity. The electrical conductivity of metal varies with the temperature. The electrical resistance of a metal, to the flow of current, is due to scattering of conduction electrons by lattice vibrations. When the temperature increases the amplitude of lattice vibrations also increases, thereby increasing the resistance. The dependence of resistance of metal (non-superconducting state) on temperature is shown in figure. The resistance decreases with temperature and reaches a minimum valueknown as residual resistance, at T = 0K. The residual resistance at T = 0K is due to impurities in the metal.



DEPENDENCE OF ρ ON T

The variation is expressed by Matthiessen's rule

$$\rho = \rho_o + \rho(T)$$

Where 'ρ' is the resistivity of the given material, 'ρ_o'is the residual resistivity and 'ρ(T)' is the temperature dependent part of resistivity called ideal resistivity.

15) Calculate the probability of an electron occupying an energy level 0.02ev above the Fermi level at 300k. (June2015)

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

$$E-E_F = 0.02eV$$

$$kT = \frac{1.38x10-23x300}{1.6x10-19} = 0.026eV$$

$$\frac{E-EF}{kT} = \frac{0.02}{0.026} = 0.77$$

$$F(E) = \frac{1}{2.16+1} = 0.32 = 32\%$$

16) Define the drift velocity, mean free path ,mean collision time and relaxation time. (June2015)

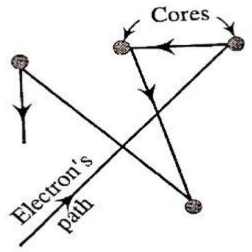
Drift velocity :v_d

"The average velocity with which free electrons move in a steady state opposite to the direction of the electric field in a metal is called ***drift velocity".***

Mean free path: (Λ)

"The average distance travelled by an electron between the successive collision during their random motion is called mean free path."

Mean collision time: (τ)



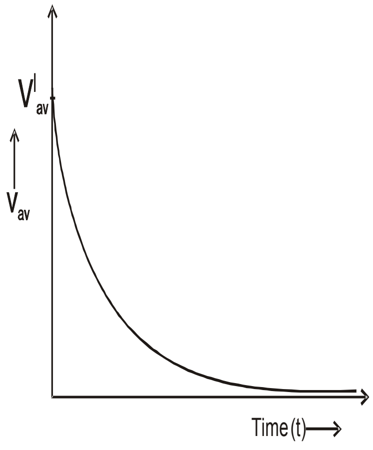
"The average time that elapses between two consecutive collisions of an electron with the lattice points is mean collision time." It is given as

$\tau = ^{\Lambda}$ Where Λ - mean free path

v -total velocity of the electrons due to the combined effect of thermal and drift velocities.

Relaxation time: (τ_r)

In the absence of an applied field, the average velocity of the electrons is zero and has some finite value under an applied field.



v_{av} = o in the absence of the field.

$v_{av} = v'_{av}$ in the presence of electric field.

If the field is turned off suddenly, the average velocity v_{av} reduces exponentially to zero.

Mathematically

$v_{av} = v'_{av}\ e^{-t/}\tau_r$

The time 't' is counted from the instant the field is switched off and τ_r is a constant, called *relaxation time.*

If $t = \tau_r$

Then $v_{av} = v'_{av} e^{-1}$

or

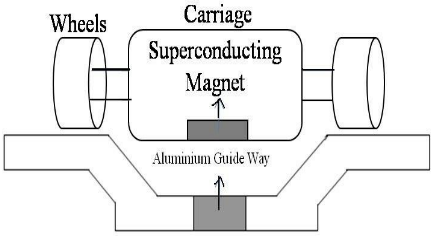
$$\boxed{v_{av} = \frac{1}{e} v'_{av}}$$

*"Thus, the **relaxation time** (τ_r) is defined as the time during which the average velocity (v_{av}) decreases to $\frac{1}{e}$ times its value at the time when the field is turned off".*

16) Describe Maglev vehicle. (June2015)

Magnetically leviated vehicles are called Maglev vehicles. The magnetic leviation is based on the principle of Meissner effect. The repulsion between the two strong fields produced by the superconducting magnet and electric currentlevitates the vehicleand is set afloat. The afloating of the vehicles by this principle is called *Magnetic Levitation.*

The superconducting magnet is kept inside the vehicle and the electric current flows in the Aluminium guide way. Once the magnetic field in the vehicle is switched on is repelled by the magnetic field of the guide way. As it is about to levitate, the guide way propels the vehicle forward by the segments provided in the Aluminium guide way. The vehicle is provided with retractable wheels. Initially the vehicle runs on the guide way, once it is levitated in air the wheels are retracted into the body. The height to which the vehicle is levitated above guide way is about 10 to 15cm. While stopping, the wheels are drawn out and the vehicle slowly settles on the guideway by running a distance. Since such vehicles float under magnetic effect, they do not have friction, less power consumption, and noiseless travel.



17) Calculate he concentration a which the acceptor atoms must be added to a germanium sample to p-type semiconductor with conductivity 0.15 per ohm-meter. Given the mobility of holes=0.17m^2/Vs (June2015)

$\Sigma = N_h e\mu_h$

$N_h = \frac{\sigma_h}{e\mu_h} = \frac{0.15}{1.625X10^{-19}X0.17} = 5.5X10^{18}$

Module-III

Laser and Optical fibers

1). Obtain the expression for energy density of radiation under equilibrium condition in terms of Einstein coefficient. (Jan2013)

Consider two energy states E_1 and E_2 of a system of atoms. Let N_1 and N_2 be the no of atoms (no.density population) in the energy states E_1 and E_2 respectively.

(Let Uν be the energy incident / unit volume). Let radiations of wavelength in the range λ and λ + dλ be incident on the system of atoms.

Let be Uν the energy incident / unit volume of the system. Then Uνdν represents the energy density of frequency v let us consider.

1.The Case of Induced absorption:

In the case of induced absorption (when) an atom in the ground state. When absorbs a radiation of frequency v = $\frac{E_2 - E_1}{h}$ then it makes transients the level E_2.

No.of such absorption per unit time per unit volume is called Rate of absorption.

The rate of absorption = $B_{12}N_1U\nu$.

Where B_{12} is the proportionality constant called Einstein coefficient of induced absorption.

2. Spontaneous Emission:

An atom in the excited state E_2 makes transition to ground state E_1 by the emission of photon of frequency v. No.of such spontaneous emissions per unit time per unit volume is called Rate of spontaneous emission. Which is proportional to the no. density in the higher energy state N_2.

The rate of spontaneous emission α N_2

Rate of spontaneous emission = $A_{21}N_2U\nu$.

Where A_{21} is called Einstein coefficient of spontaneous emission.

3.Stimulated emission:

When a radiation of frequency v incident on an atom in the state E_2. Which stimulate the atom to make downward transitionand causes the emission of stimulated photons.

The rate of stimulated emission ᴀ N_2.

Rate of stimulated emission = $B_{21}U_vN_2$

Where B_{21} is the constant of proportionality called the Einstein coefficient of stimulated emission. And N_2 is the No, density in the higher energy state E_2. At thermal equilibrium.

At thermal equilibrium,

Rate of absorption = (Rate of spontaneous emission + Rate of stimulated emission)

$B_{12}N_1U_N = A_{21}N_2 + B_{21}N_2U_N$

$U_N (B_{12}N_1 - B_{21}N_2) = A_{21}N_2$

$U_N = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$

i.e $U_N = \frac{A_{21}}{B_{21}} \left[\frac{N_2}{\frac{B_{12}}{B_{21}}N_1 - N_2} \right]$

$= \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}N_1}{B_{21}N_2} - 1} \right]$ → (1)

By Boltzmann's law,

$N_2 = N_1 e^{-\left(\frac{E_2 - E_1}{kT}\right)} = N_1 e^{-hN/KT}$

i.e., $N_1/N_2 = e^{hN/KT}$

Eqn. (1) becomes $$U_Y = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\left(\frac{hY}{kT}\right)} - 1} \right]$$ → (2)

By Planck's law, $$U_Y = \frac{8\Pi h Y^3}{c^3} \left[\frac{1}{e^{\left(\frac{hY}{kT}\right)} - 1} \right]$$ → (3)

Comparing equation (2) & (3)

$\frac{A_{21}}{B_{21}} = 8\pi hN^3/c^3$ & $\frac{B_{12}}{B_{21}} = 1$ i.e. $B_{12} = B_{21}$

The probability of induced adsorption is equal to the stimulated emission.

Because of the of identity ,the subscripts could be dropped and A_{21}&B_{21} can be represented as A & B. Therefore the equation for energy density at thermal equilibrium is ,

$$U_Y = \frac{A}{B} \left[\frac{1}{e^{\left(\frac{hY}{kT}\right)} - 1} \right]$$

Conclusions of Einstein co-efficient:

Dependence of nature of emission on frequency:

Consider $\frac{A_{21}}{B_{21}} = 8\pi hN^3/c^3$ → (1)

If A_{21} has high value, the probability of spontaneous emission is high. If B_{21} has high value, the probability of stimulated emission is high.

Further $\frac{A_{21}}{B_{21}}$ **α** N^3

Since $N = \Delta E/h$, in normal condition, the energy difference between the two levels E_1 and E_2 is large

$\frac{A_{21}}{B_{21}} >> 1$ or $A_{21} >> B_{21}$

Thus the probability of spontaneous emission is more than the stimulated emission

2). What is holography?Explainprinciple of hologram recording using laser.

(Jan2013)(July 2014)

Holography is the technique of producing 3-dimentional image of an object on 2-dimentional recording aid, by the phenomenon of interference. A and B are two identical or coherent beams incident on photosensitive surface at different angles. Due to interference effect, interference fringes are recorded on developing the photographic plate.

Principle of Hologram construction:

Light wave reflected from an object are characterized by their intensity (square of amplitude) and phase. When both intensity and phase attributes of the wave coming from three dimensional object is recorded on a photographic plate, it is called construction of hologram. When recorded photographic plate (hologram) is illuminated by a coherent light source, the three dimensional image of the original object is formed. This formation of image is known as reconstruction process.

Recording phase variation in a Hologram:

In recording hologram of an object a photographic plate is placed in front of an object at a suitable distance. Consider a coherent light incident on the object. The light reflected from two nearby points on the object travel slightly different distances in reaching the photographic plate due to variation in depth on the object. Thus the two wave fronts arrive at the photographic plate in a slightly different phase. Hence the light reflected from different points on the object will have different phases and interfere with the reference beam. The fringes recorded in the hologram carry information regarding the phase difference.

In holography there are two phases:

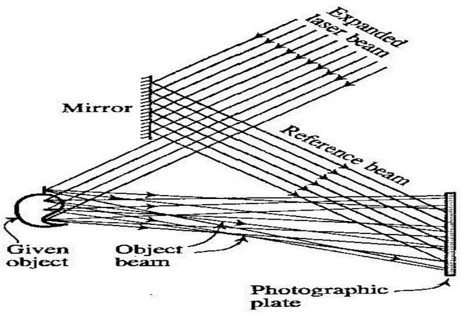
1) Recording
2) Reconstruction of the image.

Recording has two methods

1) Wave front division technique.
2) Amplitude division technique.

1) Recording of the image of an object by wave front division technique

Expanded coherent laser beam from the laser source is obtained. A portion of it is made to incident on the mirror and other portion is made to incident on the object as shown in the fig.



HOLOGRAM RECORDING

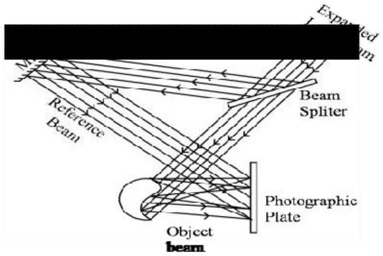
GABOR ZONE PLATE

Photographic plate is placed at a suitable position so that it receives the light reflected from both the mirror and the object. The light reflected from the mirror form a plane wavefront. It is called reference beam. The light reflected from each point on the object form a spherical wavefront. It is called object beam. Thus the interference effects of the two beams are recorded on the photographic plate.

As the spherical wave intersect the plane wave in circular zones, the interference pattern consists of concentric circular rings having constructive and destructive interference. It is called *Gabor Zone plate.*

Hologram consists of number of such zone plates. The centre of each is displaced from the other. In the recorded pattern the neighbouring zones overlap each other and become apparent, once the film is developed. It is called a hologram.

2) *Recording the image of an object by amplitude division technique:*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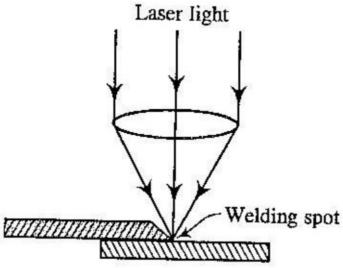
Expanded coherent laser beam from the laser source is obtained. It is made to incident on the beam splitter. The beam splitter reflects the portion of the light which is incident on the mirror. The transmitted light from the beam splitter is incident on the object. The reflected plane wavefront from the mirror and reflected spherical wavefronts from different points on the object undergoes interference on the photographic plate kept at a suitable place. The interference fringes are recorded on the photographic plate. The developed photographic plate becomes the hologram of the object.

3).Explainlaser welding and cutting process with diagrams.(July2013)

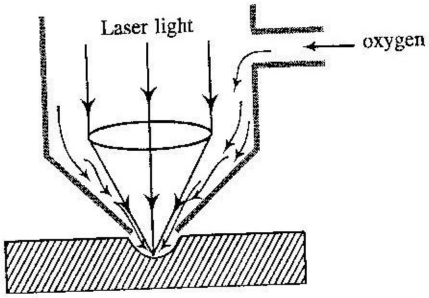
1. ***Laser welding***: Laser beam is allowed to fall on the surface to be welded. The beam is absorbed and the surface starts melting due to the heat generated. The reflectivity and thermal diffusivity of the material are the important factors of welding. The reflectivity is more, the energy released is less. To increase the energy, the surface is coated with either cupric oxide or graphite. During the melting, the impurities accumulate on the surface of the material and it becomes a homogeneous solid structure and forms a stronger joint on cooling.

The laser welding is very narrow and accurate. In laser welding, laser does not have any physical contact with the material to be welded. Hence no contamination occurs. The properties of the material do not change due to welding. Laser welding can be carried out in normal workshop environment.



LASER WELDING

2. ***<u>Laser cutting:</u>*** When the laser is allowed to focus onto a tiny spot, the metal gets vaporized. There will be no physical distortion. CO_2 laser is used for the laser cutting. The laser cutting depends upon the input power for the laser beam and inversely proportional to the thickness of the material. It also depends on the nature of the gas used with the laser beam. Oxygen is commonly used gas. The gas is flown coaxially with the beam. The combustion of the gas helps the metal to get vaporized and it reduces the laser power which is required for cutting. Laser cuts the material accurately.

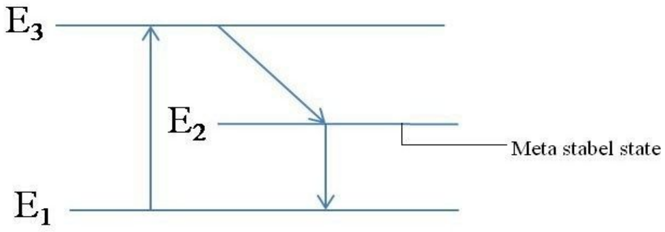


LASER CUTTING

4). Mention the condition for laser action. Explain the working of a semiconductor laser. (Dec2014)

Conditions for laser action:

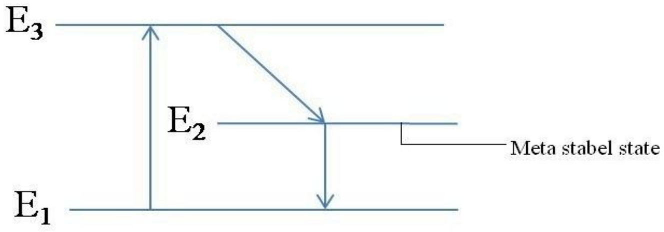
1. ***Meta Stable State:*** *It is the state in which the atoms or molecules stay for an unusually long time of the order of* 10^{-3} *to* 10^{-2} *second.*



This state plays an important role in lasing action. This property helps in achieving population inversion, In excited state other than metastable atom stay of order of 10^{-8} second.

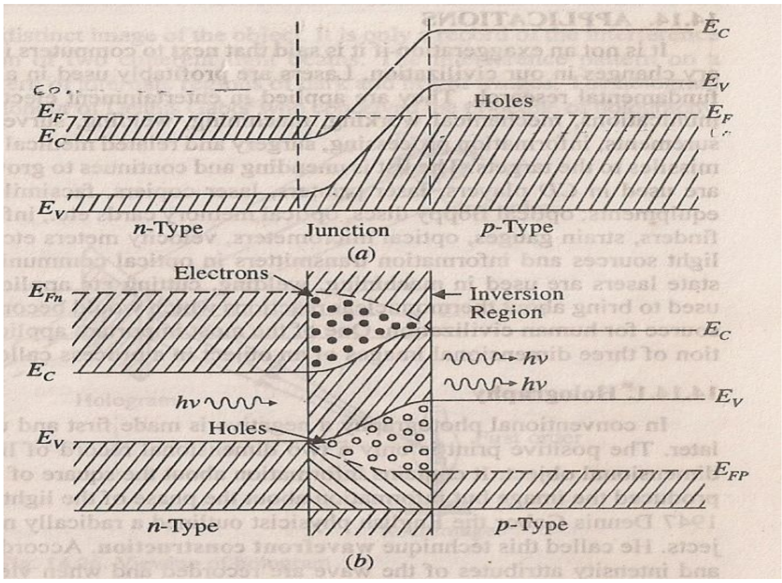
2. ***Population Inversion:*** *It is the state of the system at which the population of a particular higher energy level is made to be greater than that of the lower energy level.*

Let E_1, E_2, E_3 be the energy levels of the system $E_3>E_2>E_1$. E_2 is the metastable state of the system. Atoms get excited from the state E_1 to E_3 by means of external source and stay there for short time. These atoms undergo spontaneous transitions to E_2 and E_1. The atoms at the state E_2 stay for longer time. A stage is reached in which the number of atoms at state E_2 is more than the number of atoms at E_1 which is known as population inversion.



Working of a semiconductor laser: The energy band diagram of heavily doped pn-junction is as shown unbiased condition. At thermal equilibrium, the Fermi level is uniform across the junction.

Because of very high doping on n-side, Fermi level is pushed into the conduction band and electrons occupy the portion of the conduction band lying below the Fermi level. On P-side, the Fermi level lies within the valence band and holes occupy the portion of the valence band that lies above the Fermi level. When the junction is forward biased electrons and holes are injected into the junction region in high concentrations. At low forward current, the electron-holes recombination results in spontaneous emission of photons and the junction acts as a LED. As the forward current is increased gradually and when it reaches a threshold value the carrier concentration in the junction region there will be large concentrations of electrons within the band. As a result condition of population inversion is attained in the narrow region. This narrow zone in which population inversion occurs is called as an active region, at that stage a photon emitted spontaneously triggers stimulated emission. This stimulated electron-hole recombination produces coherent radiation.



Energy level diagram of p-n junction diode laser

(a) Before biasing (b) After biasing.

The stimulated electron-hole recombination causes emission of coherent radiation of very narrow bandwidth. At room temperature, GaAs laser emits light of wavelength $9000A^0$ and GaAsP laser radiates at $6500A^0$.

The wavelength of the emitted laser beam is

$\Lambda = \frac{hC}{E_g}$ Where h – Planck's const.

5) Discuss the various loss factors in optical fiber communication. (Dec2014)

Types of loss or attenuation in an Optical fiber are,

1. ***Absorption loss:*** Absorption of photons by impurities like metal ions such as iron, chromium, cobalt and copper in the silica glass of which the fiber is made of. During signal processing photons interact with electrons of impurity atoms. The atoms are excited and de-excite by emitting photons of different characteristics. Hence it is a loss of energy. The other impurity such as hydroxyl ions (OH) causes significant absorption loss. The absorption of photons by fiber material itself is called intrinsic absorption.

2. ***Scatteringloss*****:** When the wavelength of the photon is comparable to the size of the particle then the scattering takes place. Because of the non-uniformity in manufacturing, the refractive index changes with length leads to a scattering. This type of scattering is called as Rayleigh scattering. It is inversely proportional to the fourth power of wavelength. Scattering of photons also takes place due to trapped gas bubbles which are not dissolved at the time of manufacturing.

3. ***Radiation loss***: Radiation losses occur due to macroscopic bends and microscopic bends.

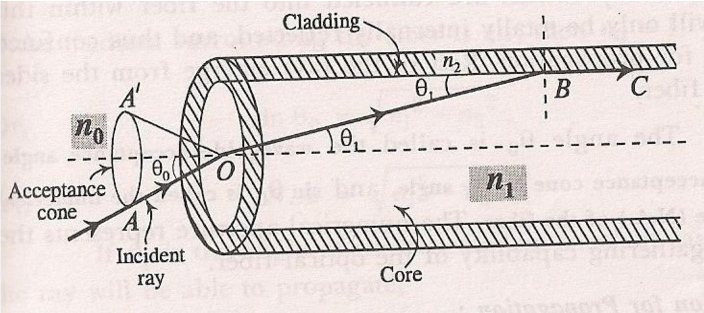
Macroscopic bending: All optical fibers are having critical radius of curvature provided by the manufacturer. If the fiber is bent below that specification of radius of curvature, the light ray incident on the core cladding interface will not satisfy the condition of TIR. This causes loss of optical power.

Microscopic bending: Optical power loss in optical fibers is due to non-uniformity of the optical fibers when they are laid. Non uniformity is due to manufacturing defects and also lateral pressure built up on the fiber. The defect due to non-uniformity (microbending) can be overcome by introducing optical fiber inside a good strengthen polyurethane jacket

6). Derive the condition for propagation of light through an optical fiber (Dec2014)

Consider a light ray AO incident at an angle ‘θ_0’ enters into the fiber. Let ‘θ_1’ be the angle of refraction for the ray OB. The refracted ray OB incident at a critical angle (90°- θ_1) at B grazes the interface between core and cladding along BC. If the angle of incidence is greater than critical angle, it undergoes total internal reflection. Thus θ_0 is called the waveguide acceptance angle and $\sin\theta_0$ is called the numerical aperture.

Let n_0, n_1 and n_2 be the refractive indices of the medium, core and cladding respectively.



ACCEPTANCE CONE

Applying Snell’s law,at ‘O’

$$n_o \sin\theta_0 = n_1 \sin\theta_1 \rightarrow (1)$$

At B the angle of incidence is $(90 - \theta_1)$

From Snell's law, at 'B'

$$n_1 \sin(90-\theta_1) = n_2 \sin 90$$

$$n_1 \cos\theta_1 = n_2$$

$$\cos\theta_1 = n_2 / n_1 \rightarrow (2)$$

From eqn (1)

$$Sin\Theta_0 = \frac{n_1}{n_0} \sin\Theta_1$$

$$= \frac{n_1}{n_0} \sqrt{1 - cos^2\theta_1} \rightarrow (3)$$

Using eqn. (2) in (3)

$$Sin\Theta_0 \quad \frac{n_1}{n_0}\sqrt{1 - \frac{n_2^2}{n_1^2}} \quad \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \rightarrow (4)$$

If the surrounding medium is air, then $n_o = 1$ & $Sin\Theta_0 \quad \sqrt{n_1^2 - n_2^2}$

Where $Sin\theta_0$ is called numerical aperture.

$$Sin\Theta_0 = N.A = \sqrt{n_1^2 - n_2^2}$$

Therefore for any angle of incidence equal to θ_i equal to or less than θ_0, the incident ray is able to propagate by TIR. ie, $\theta_i < \theta_0$ and $Sin\Theta_i < Sin\Theta_0$

$$sin\theta_i < \sqrt{n_1^2 - n_2^2}$$

$sin\theta_i < N.A$ is the condition for propagation.

7). The average power of laser beam of wavelength 6328A is 5mW.Find the number of photons emitted per second by the source. (Dec2014)

$$\Delta E = hN = \frac{hc}{\Lambda} = \frac{6.63\times 10^{-34}\times 3\times 10^{8}}{6328\times 10^{-10}} = 3.143\times 10^{-19}\ \text{J}$$

This energy difference becomes the energy of each photon.

Let N is the number photons emitted per second then

$$N \times \Delta E = P$$

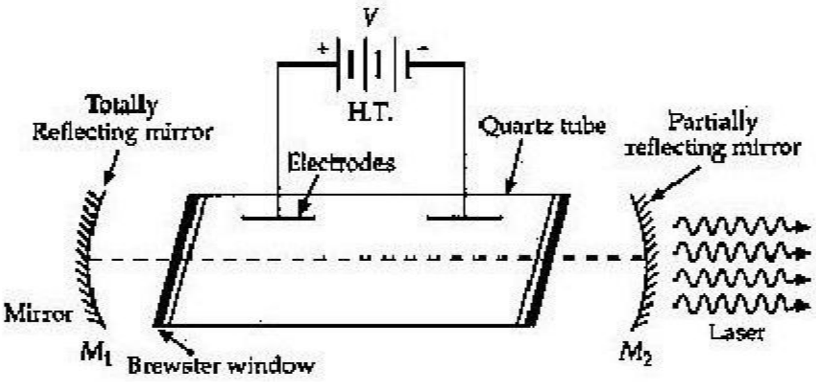
$$N = \frac{5\times 10^{-3}}{3.143\times 10^{-19}} = 3.143\times 10^{16}$$

8). What is laser? Give the construction and working of carbon dioxide laser device. (Dec2014)

The word Laser stands for ***L**ight **A**mplification by **S**timulated **E**mission of **R**adiation.* It is a device which amplifies light.

<u>Carbon dioxide (CO_2) laser:</u>

Construction: Carbon dioxide laser consists of a discharge tube having a diameter of 2.5cm and a length of about 5m. The discharge tube is filled with a mixture of carbon dioxide, nitrogen and helium gases in the ratio of 1:2:3 with traces $H_2$of water vapour. Pressures maintained are about 6 -17 torr.



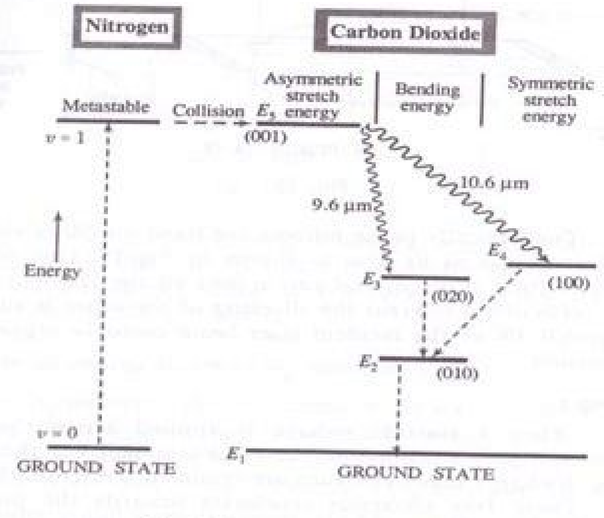
Two optically plane mirrors, one fully silvered and other one half silvered are fixed on either side of the tube normal to its axis. There is 100% reflectivity from the fully silvered mirror and polarized laser light comes out from the half silvered mirror.

Working:

When a suitable voltage is applied across the two electrodes a glow discharge of the gasses is initiated in tube .During the discharge many electrons are set free from gas atoms and move towards positive electrode at which time they begin to collide with $N_2$and O_2 molecules in their path, which excites N_2 molecules are excited to the first vibrational level V=1 which is a metastable state of N_2molecule

$$\mathbf{e_1+N_2 \text{ ----}> e_2 + N_2^*}$$

Where $\mathbf{e_1}$ and $\mathbf{e_2}$are the energy values of the electrons before and after the collision respectively, and $\mathbf{N_2}$and $\mathbf{N_2}^*$are the energy values the $\mathbf{N_2}$molecules in the ground state ie, V=0 & V=1 state.



CO_2 LASER ENERGY LEVEL DIAGRAM

Likewise, many of the CO_2 molecules will also be raised to their (001) energy state (which is not metastable state).For $\mathbf{N_2}$molecules since V =1 is the metastable state ,the molecules remain there for relatively long time which leads to increase of population in V=1 state. So that when $\mathbf{N_2}$molecule in the metastable state collides with CO_2 molecules in the ground state ,because of matching of energy levels, resonant energy transfer takes place from the $\mathbf{N_2}$ to CO_2 molecule which elevated to the (001) state whereas ,N_2molecule returns to the ground state .

Ie $\mathbf{N_2}^*+CO_2 \text{ -}> \mathbf{N_2}+CO_2^*$

Where ,$CO_2$and CO_2* are energies of CO_2molecule in the ground and excited state respectively.

Thus the population of the (001) or E_5 level of CO_2increases rapidly which leads the population inversion. Let us denotes ground state,(010)(020)(100)and(001) states as E_1,E_2,E_3,E_4 and E_5 levels respectively as shown in figure. Once the population inversion is established between E_5level with respect to the E_3& E_4 levels then two laser transition takes place within CO_2 gas.

Those transitions are ,

1. Transition from E5 to E4 which gives the radiation of wavelength 10.6µm,which is comes under far infrared(IR) region.

2. Another transition from E5 to E3 level which gives rise to radiation of wavelength 9.6μm which is also comes under far IR region.

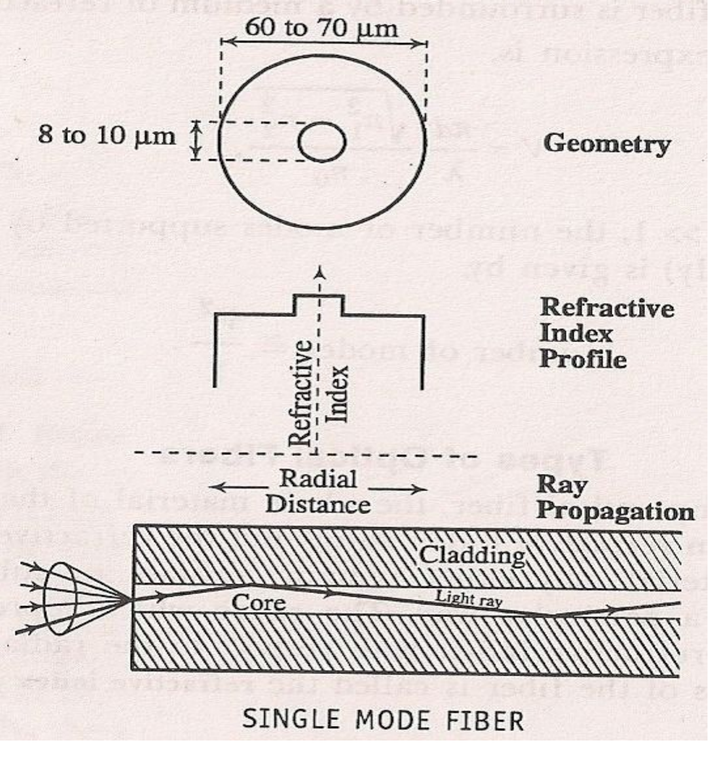
The CO_2 molecules in the E2 level undergo collision with He & H_2O vapour molecules efficiently & come down to the ground state.CO_2 laser operates with an efficiently of 30% power output of few kilo watts can be continuously maintained in a medium.

9). What are the different types of optical fibers? Explain. (Dec2014)

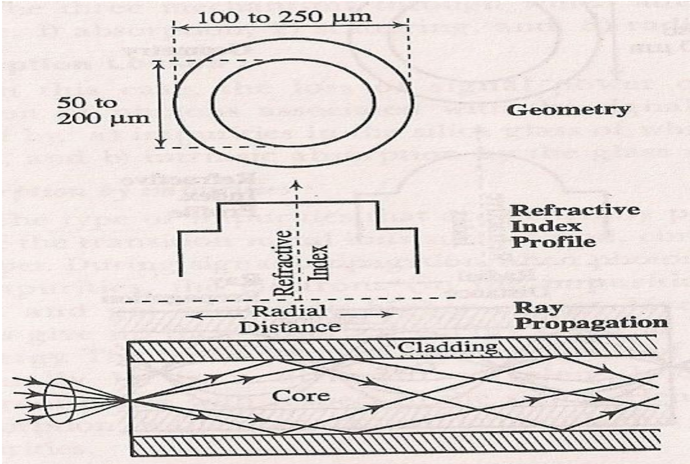
Following are the different types of fibers:

1. Single mode fiber
2. Step index multimode fiber
3. Graded index multimode fiber(GRIN)

Single mode fiber**:** Refractive index of core and cladding has uniform value; there is an increase in refractive index from cladding to core. Thus its refractive index profile takes the shape of a step. The core diameter is about 8 to 10 micro meter and external diameter of cladding 60 to 70 micrometer. Because of its narrow core,it can guide just a single mode and hence there is no intermodal dispersion. They are used in submarine cabal system.

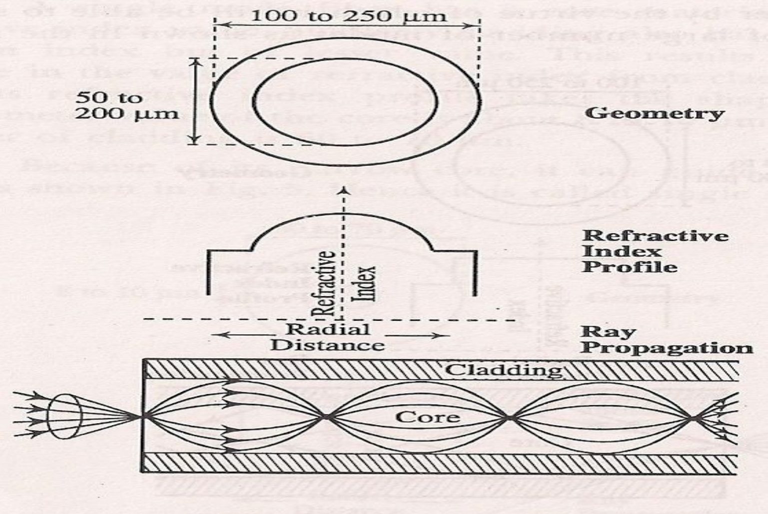


2. ***Step index multimode fiber:*** It is similar to single mode fiber but core has large diameter. It can propagate large number of modes as shown in figure. Laser or LED is used as a source of light. It has an application in data links.



STEP - INDEX MULTIMODE FIBER

3. ***Graded index multimode fiber:*** It is also called GRIN. The refractive index of core decreases from the axis towards the core cladding interface. The refractive index profile is shown in figure. The incident rays bends and takes a periodic path along the axis. The rays have different paths with same period. Laser or LED is used as a source of light. It is the expensive of all. It is used in telephone trunk between central offices.



GRADED - INDEX MULTIMODE FIBER

10).The attenuation in an optical fiber is 3.6dB/km. What fraction of its initial intensity remains after 3km. (Dec2014)

Data given, A =3.6 dB/km.

$\left(\frac{p_{out}}{p_{in}}\right)$ =? At the end of 3km,L=3km

Solution:

$$A = -\frac{10}{L}\log_{10}\left(\frac{p_{out}}{p_{in}}\right) \text{ dB/km}$$

$$\log_{10}\left(\frac{p_{out}}{p_{in}}\right) = \frac{AL}{10}$$

At the end of 3km , $\left(\frac{p_{out}}{p_{in}}\right) = 10^{\frac{-3.3\times 3}{10}} = 0.0832$

At the end of 3km the fraction of intensity of light that is available will be 0.0832(8.3%).

11). Derive an expression for the radiant energy density under thermal equilibrium using Einstein's coefficients. (June 2015)

Consider two energy states E_1 and E_2 of a system of atoms. Let N_1 and N_2 be the no of atom (no.density population) in the energy states E_1 and E_2 respectively.

(Let Uν be the energy incident / unit volume). Let radiations of wavelength in the range λ and λ + dλ be incident on the system of atoms.

Let be Uν the energy incident / unit volume of the system. Then Uνdν represents the energy density of frequency v let us consider.

i) The Case of Induced absorption:

In the case of induced absorption (when) an atom in the ground state. When absorbs a radiation of frequency v = $\frac{E_2 - E_1}{h}$ then it makes transients the level E_2.

No.of such absorption per unit time per unit volume is called Rate of absorption.

The rate of absorption = $B_{12}N_1$ Uν.

Where B_{12} is the proportionality constant called Einstein coefficient of induced absorption.

ii) Spontaneous Emission:

An atom in the excited state E_2 makes transition to ground state E_1 by the emission of photon of frequency v. No.of such spontaneous emissions per unit time per unit volume is called Rate of spontaneous emission. Which is proportional to the no. density in the higher energy state N_2.

The rate of spontaneous emission α N_2

Rate of spontaneous emission = $A_{21}N_2$ Uν.

Where A_{21} is called Einstein coefficient of spontaneous emission.

iii) Stimulated emission:

When a radiation of frequency v incident on an atom in the state E_2. Which stimulate the atom to make downward transition and causes the emission of stimulated photons.

The rate of stimulated emission A N_2.

Rate of stimulated emission = $B_{21}U_\nu N_2$

Where B_{21} is the constant of proportionality called the Einstein coefficient of stimulated emission.

And N_2 is the No, density in the higher energy state E_2. At thermal equilibrium.

At thermal equilibrium,

Rate of absorption = (Rate of spontaneous emission + Rate of stimulated emission)

$$B_{12}N_1U_N = A_{21}N_2 + B_{21}N_2U_N$$

$$U_N (B_{12}N_1 - B_{21}N_2) = A_{21}N_2$$

$$U_N = \frac{A_{21}N_2}{(B_{12}N_1 - B_{21}N_2)}$$

i.e $$U_N = \frac{A_{21}}{B_{21}} \left[\frac{N_2}{\left(\frac{B_{12}}{B_{21}}N_1 - N_2\right)}\right]$$

$$= \frac{A_{21}}{B_{21}} \left[\frac{1}{\left(\frac{B_{12}N_1}{B_{21}N_2}\right) - 1}\right] \rightarrow (1)$$

By Boltzmann's law,

$$N_2 = N_1 e^{-\left(\frac{E_2 - E_1}{KT}\right)} = N_1 e^{-hN/KT}$$

i.e., $N_1/N_2 = e^{hN/KT}$

Eqn. (1) becomes $$U_Y = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{hY/KT} - 1}\right] \rightarrow (2)$$

By Planck's law, $$U_\nu = \frac{8\pi h \nu^3}{c^3}\left(\frac{1}{e^{\left(\frac{h\nu}{kT}\right)}-1}\right) \rightarrow (3)$$

Comparing equation (2) & (3)

$\frac{A_{21}}{B_{21}} = 8\pi h N^3/c^3$ & $\frac{B_{12}}{B_{21}} = 1$ i.e. $B_{12} = B_{21}$

The probability of induced adsorption is equal to the stimulated emission.

Because of the of identity ,the subscripts could be dropped and A_{21}&B_{21} can be represented as A & B. Therefore the equation for energy density at thermal equilibrium is ,

$$U_\nu = \frac{A}{B}\left(\frac{1}{e^{\left(\frac{h\nu}{kT}\right)}-1}\right)$$

12) With suitable ray-diagrams, explain the principle construction of a holographic images. (June 2015)

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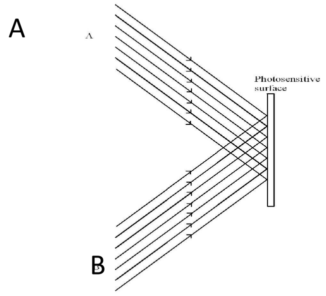

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When recorded photographic plate (hologram) is illuminated by a coherent light source, the three dimensional image of the original object is formed. This formation of image is known as reconstruction process.

In holography there are two phases:

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4) Reconstruction of the image.

Recording has two methods

3) Wave front division technique.
4) Amplitude division technique.

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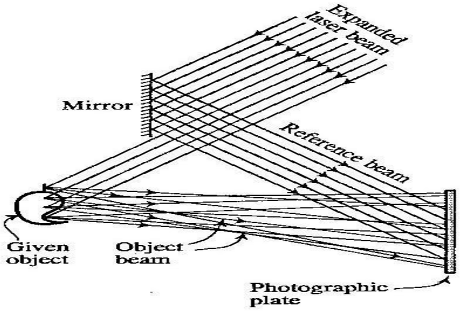


HOLOGRAM RECORDING

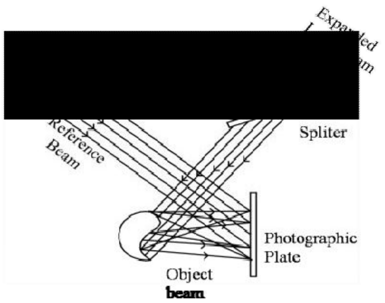
GABOR ZONE PLATE

Photographic plate is placed at a suitable position so that it receives the light reflected from both the mirror and the object. The light reflected from the mirror form a plane wavefront. It is called reference beam. The light reflected from each point on the object form a spherical wavefront. It is called object beam. Thus the interference effects of the two beams are recorded on the photographic plate.

As the spherical wave intersect the plane wave in circular zones, the interference pattern consists of concentric circular rings having constructive and destructive interference. It is called *Gabor Zone plate.*

Hologram consists of number of such zone plates. The centre of each is displaced from the other. In the recorded pattern the neighbouring zones overlap each other and become apparent, once the film is developed. It is called a hologram.

4) Recording the image of an object by amplitude division technique: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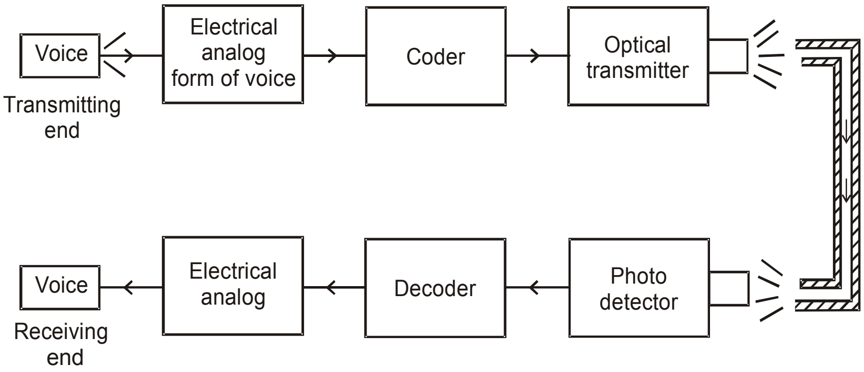
Expanded coherent laser beam from the laser source is obtained. It is made to incident on the beam splitter. The beam splitter reflects the portion of the light which is incident on the mirror. The transmitted light from the beam splitter is incident on the object. The reflected plane wavefront from the mirror and reflected spherical wavefronts from different points on the object undergoes interference on the photographic plate kept at a suitable place. The interference fringes are recorded on the photographic plate. The developed photographic plate becomes the hologram of the object.

13). Give an account of point to point communication system using optical fibers.

(June 2015)

(Point to point communication in OF)



Point of Point optical fibre communication system

A fiber optic communication system is very much similar to a traditional communication system and has three major components. A transmitter converts electrical signals to light signals, an optical fibre transmits the signals and a receiver captures the signals at the other end of the fiber and converts them to electrical signals.

The transmitter consists of a light source supported by necessary drive circuits. First voice is converted into electrical signals using a transducer. It is digitized (converted to binary electrical signals) using a coder. The digitized signal, which carries the voice information, is fed to an optical transmitter. The light source in optical transmitter (LED or laser diode) emits modulated light, which is transmitted through optical fiber. The light emitted by the source is in the IR range with a wavelength of 850nm, 1300nm or 1550nm.

On the other end the modulated light signal is detected by a photo detector is amplified and is decoded using a decoder. The output is fed to a suitable transducers to convert it into an audio or video form.

14) The angle of acceptance of an optical fiber kept in air is 30□. Find the angle of acceptance when the fiber is in a medium of refractive index 4/3. (June 2015)

Solution:

$$\mathrm{Sin}\theta' = \frac{\mathrm{Sin}\theta}{n_0} = \frac{\sin 30^\circ}{\frac{4}{3}}$$

$$\theta' = 22^\circ$$

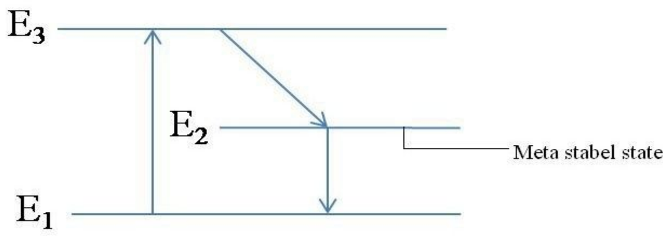
15) Discuss the requisites and the conditions for a laser system. (June 2015)

<u>Requisites of a Laser System:</u>

1) The excitation source for pumping action: *It is the external source which provides energy in an appropriate form for pumping the atoms to higher energy levels.*
2) Active medium: The *quantum system in which the pumping &lasing actions occur, is called an active system.*
3) Laser cavity: Is an active medium bounded by two mirrors. (Resonator)

<u>Condition for laser action:</u>

3. ***<u>Meta Stable State:</u>****It is the state in which the atoms or molecules stay for an unusually long time of the order of* 10^{-3} *to* 10^{-2} *second.*

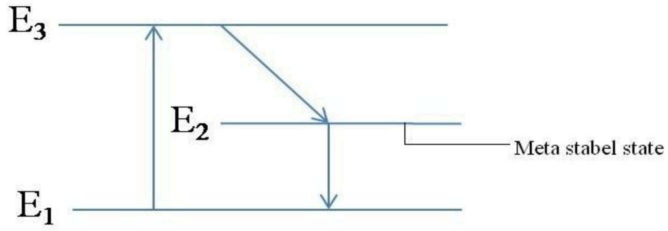


This state plays an important role in lasing action. This property helps in achieving population inversion, In excited state other than metastable atom stay of order of 10^{-8} second.

4. ***<u>Population Inversion:</u>****It is the state of the system at which the population of a particular higher energy level is made to be greater than that of the lower energy level.*

Let E_1, E_2, E_3 be the energy levels of the system $E_3>E_2>E_1$. E_2 is the metastable state of the system. Atoms get excited from the state E_1 to E_3 by means of external source and stay there for short time. These atoms undergo spontaneous transitions to E_2 and E_1. The atoms at the state E_2

stay for longer time. A stage is reached in which the number of atoms at state E_2 is more than the number of atoms at E_1 which is known as population inversion.



16) Define angle of acceptance and numerical aperture. Obtain an expression for the numerical aperture of an optical fiber. (June 2015)

Angle of acceptance is the angle of incidence for which the refracted ray grazes the core-cladding interface of an optical fiber.

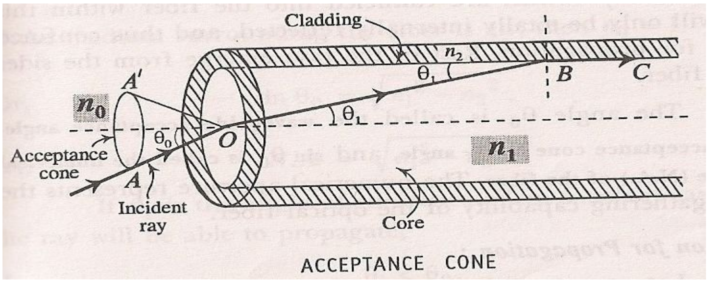
Numerical aperture is the light gathering ability of an optical fiber.

Consider a light ray AO incident at an angle 'θ_0' enters into the fiber. Let 'θ_1' be the angle of refraction for the ray OB. The refracted ray OB incident at a critical angle $(90^\circ - \theta_1)$ at B grazes the interface between core and cladding along BC. If the angle of incidence is greater than critical angle, it undergoes total internal reflection. Thus θ_0 is called the waveguide acceptance angle and $\sin\theta_0$ is called the numerical aperture.

Let n_0, n_1 and n_2 be the refractive indices of the medium, core and cladding respectively.

From Snell's law,

$$n_o \sin\theta_0 = n_1 \sin\theta_1 \rightarrow (1)$$



ACCEPTANCE CONE

From Snell's law,At B the angle of incidence is $(90 - \theta_1)$

$$n_1 \sin(90-\theta_1) = n_2 \sin 90$$

$$n_1 \cos\theta_1 = n_2$$

$$\cos\theta_1 = n_2 / n_1 \rightarrow (2)$$

From eqn (1)

$$Sin\Theta_0 = \frac{n_1}{n_0}\sin\Theta_1$$

$$= \frac{n_1}{n_0}\sqrt{1 - cos^2\theta_1} \rightarrow (3)$$

Using eqn (2) in (3)

$$Sin\Theta_0 = \frac{n_1}{n_0}\sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$= \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \rightarrow (4)$$

The surrounding medium is air, $n_o = 1$

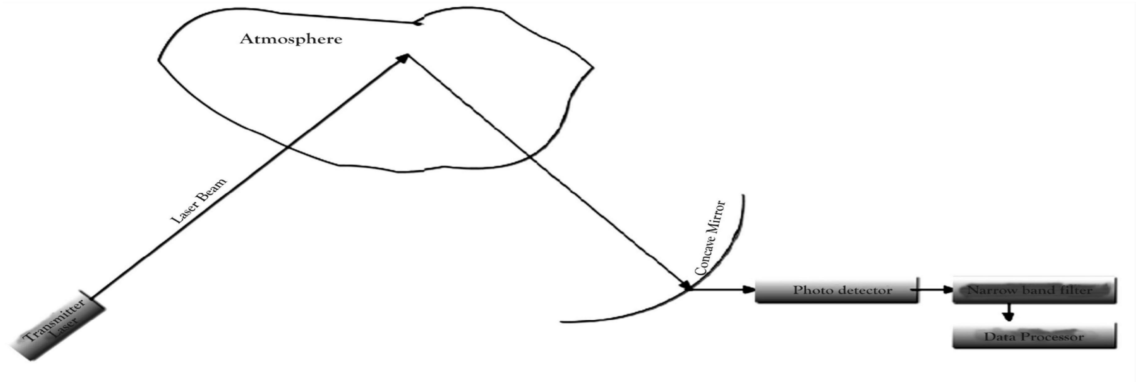
$$Sin\Theta_0 = \sqrt{n_1^2 - n_2^2}$$

Where $Sin\theta_0$ is called numerical aperture.

$$N.A = \sqrt{n_1^2 - n_2^2}$$

17) Explain measurement of pollutants in atmosphere using laser. (June 2015)

3. ***Measurement of pollutants in the atmosphere:*** In the application of laser for measurement of pollutant, laser is made use of the way as RADAR (Radio Detection and Ranging) system is used. Hence it is often referred as a LIDAR (Light Detection and Ranging) technique. In the LIDAR system, the transmitting part consists of pulsed laser and the receiving part consists of a concave mirror, photo detector and a data processor.

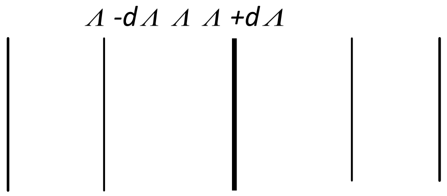


When measurements are carried out, the laser beam undergoes scattering at places in the atmosphere where there is congestion due to higher concentration of particulate matter. The back scattered light is received by the concave mirror. The distance of congestion from the measuring station is calculated on the basis of time delay between the pulse emission and reception of the back scattered light. By scanning the space around the station, the concentration of pollutants can be mapped for different vertical section of the atmosphere. This method cannot provide any information regarding the nature of the scattering particles. As this technique doesn't provide the information regarding the nature of the scattering particles following two more techniques are followed.

1. *Absorption Technique*:

The laser beam is passed through the sample collected from the atmosphere. The transmitted beam is recorded with a detector. While the beam passes through the sample, it undergoes absorption of various degrees depending upon the presence of exact type of chemical substance that the particulate comprise of. Depending upon the characteristic absorption pattern observed in the recording, the composition of the atmospheric pollutants could be determined.

2.*Raman Back Scattering*: Since laser is highly monochromatic, we expect to see only one line in the spectrum. But due to Raman scattering in the spectrum, several lines of low intensity will be seen symmetrically on either side of the incident line. There wavelength values will be close to that of the incident light. These additional spectral lines are called as side bands. Different gasses produce different side bands. The shifts in frequencies are termed as Raman shifts. Thus by observing the Ramanspectrum of the back scattered light in the gas sample one can assess the composition of the pollutants.



Raman Back Scattered Spectrum

18) A 5w pulsed laser emits light of wavelength 694nm. If the duration of each pulse is 20ns, Calculate the number of photons emitted per pulse. (June 2015)

Solution:

$E = \frac{N?c}{\lambda} = pt$

$\therefore N = \frac{pt\lambda}{?c} = \frac{5\times 694\times 10^{-9}\times 20\times 10^{-9}}{6.625\times 10^{-34}\times 3\times 10^{8}}$

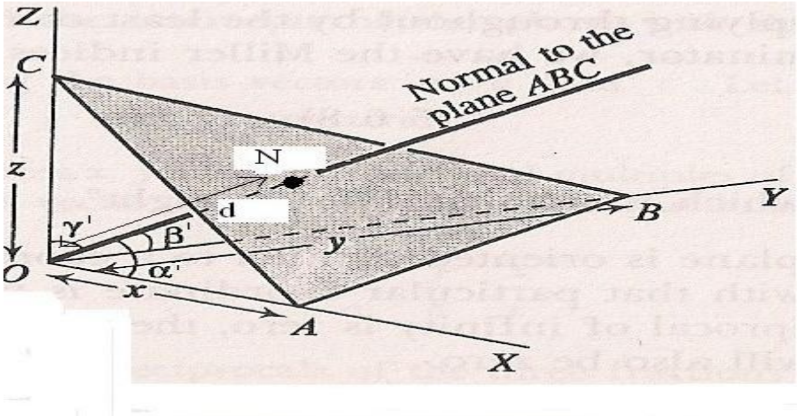
$N = 3.517\times 10^{11}$ atoms/ m^3

Module- IV

Crystal Structure

1)Derive an expression for inter planar spacing of crystal in terms of miller indices. (Jan2013)

Consider a plane ABC which belongs to a family of planes. h, k, l are the Miller indices of this plane, which represents the set of planes. The perpendicular ON from the origin O to the plane represents the inter-planar spacing d=ON of this family of planes. Let ON make an angle α^1, β^1, γ^1 with the x, y, z axes respectively.



INTERPLANAR SPACING

The intercepts of the plane on the three axes are

OA= a/h ; OB= a/k ; OC=a/l

Where ‘a’ is the length of the cube edge.

Then from figure, we have

$$\cos A^{1} = \frac{d}{OA} = \frac{d}{a/h} = \frac{dh}{a}$$

$$\cos B^{1} = \frac{d}{OB} = \frac{d}{a/k} = \frac{dk}{a}$$

$$\cos\Gamma^1 \quad \frac{d}{OC}=\frac{d}{a/l}=\frac{dl}{a}$$

From the figure $ON=\left[x^2+y^2+z^2\right]^{\frac{1}{2}}$

$$d=\left[d^2\cos^2\left(A^1\right)+d^2\cos^2\left(B^1\right)+d^2\cos^2\left(\Gamma^1\right)\right]^{\frac{1}{2}}$$

But, $\left[\cos^2\left(A^1\right)+\cos^2\left(B^1\right)+\cos^2\left(\Gamma^1\right)\right]=1$ for the orthogonal coordinates

Substituting the values of cosA^1 ,cosB^1, cosΓ^1 in equation (2),

We get,

$$\left(\frac{d}{OA}\right)^2+\left(\frac{d}{OB}\right)^2+\left(\frac{d}{OC}\right)^2=1$$

$$\left(\frac{dh}{a}\right)^2+\left(\frac{dk}{a}\right)^2+\left(\frac{dl}{a}\right)^2=1$$

$$\frac{d^2}{a^2}\left(h^2+k^2+l^2\right)=1$$

$$d=\frac{a}{\sqrt{\left(h^2+k^2+l^2\right)}}$$

This is the relation between inter-planar spacing'd' and the edge of the cube 'a'. It should be noted that this formula is applicable only to primitive lattices in cubic, orthorhombic and tetragonal systems.

2). What is atomic packing factor?Calculate packing factor for sc and bcc structures.
(Jan2013)(July2014)

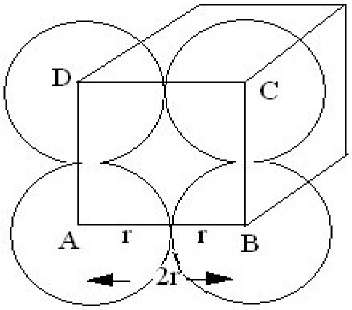
Atomic packing factor (APF)

The fraction of the space occupied by atoms in a unit cell is known as atomic packing factor. It is the ratio of the total volume occupied by the atoms in the unit cell to the total available volume of the unit cell.

Simple cubic structure (SC):

There is only one lattice point at each of the eight corners of the unit cell. In a simple cubic structure an atom is surrounded by six equidistant neighbors. Hence the co-ordination number is 6. Since each atom in the corner is shared by 8 unit cells, the total number of atoms in one unit cell is (1/8) ×8 = 1

The nearest neighbor distance ‘2r’ is the distance between the centers of two nearest neighboring atoms.



The nearest neighbor distance 2r = a

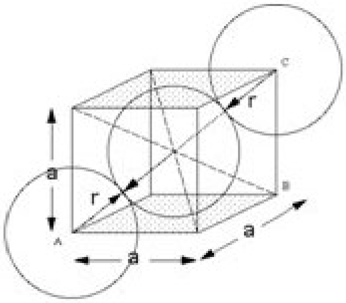
The number of lattice points per unit cell = 1

Volume of all the atoms in a unit cell $v = \frac{4}{3}\pi r^3$

Volume of unit cell = $V = a^3 = (2r)^3$

Packing factor is $P.F = \frac{v}{V} = \frac{4\Pi r^3}{3 \times 8r^3} = \frac{\Pi}{6} = 0.52 = 52\%$

Body centered cubic structure (BCC):



In a BCC structure eight atoms are present at eight corners and one atom is at the center. The co-ordination number is 8. The number of atoms per unit cell is = [(1/8)×8]+1 = 2

The lattice constant is

$(AB)^2 = a^2 + a^2 = 2a^2$

$(AC)^2 = (AB)^2 + (BC)^2$

$(4r)^2 = 2a^2 + a^2 = 3a^2$

Lattice constant $a = \frac{4r}{\sqrt{3}}$

Volume of all the atoms per unit cell v = $2\times\left(\frac{4}{3}\Pi r^3\right)$

Volume of the unit cell V = $a^3 = \frac{64r^3}{3\sqrt{3}}$

Atomic packing factor = $\frac{v}{V} = \frac{8\Pi r^3 \times 3\sqrt{3}}{3\times 64r^3} = \frac{\sqrt{3}\,\Pi}{8} = 0.68 = 68\%$

3). What are miller indices? Explain how axial intercepts in a crystal plane are converted into miller indices. (July2013) (Dec2014)

"Miller indices is a set of three lowest possible integers whose ratio taken in order is the same as that of reciprocals of the intercepts of the planes on the corresponding axes in the same order".

Procedure to find miller indices of planes:

1. Find the intercepts of the planes along x, y and z axis
2. Express the intercepts as fractional multiples of lattice constants a, b & c respectively.
3. Find out the reciprocal of these numbers.
4. Find the least common multiple (LCM) of the denominator and multiply each term with LCM.
5. The result is in the form of h, k, l. are called miller indices, denoted by (h k l)

Example:

Let the planes have intercepts 4, 1, 2

1. x:y:z=4:1:2
2. Reciprocals $\frac{1}{4}:\frac{1}{1}:\frac{1}{2}$
3. LCM=4
4. $4\times\frac{1}{4} : 4\times 1 : 4\times\frac{1}{2}$

The miller indices are (hkl) = (1 4 2)

4). What are miller indices? Explain how axial intercepts in a crystal plane are converted into miller indices. (Dec2014)

"Miller indices is a set of three lowest possible integers whose ratio taken in order is the same as that of reciprocals of the intercepts of the planes on the corresponding axes in the same order".

Procedure to find miller indices of planes:

6. Find the intercepts of the planes along x, y and z axis
7. Express the intercepts as fractional multiples of lattice constants a, b & c respectively.
8. Find out the reciprocal of these numbers.
9. Find the least common multiple (LCM) of the denominator and multiply each term with LCM.
10. The result is in the form of h, k, l. are called miller indices, denoted by (h k l)

Example:

Let the planes have intercepts 4, 1, 2

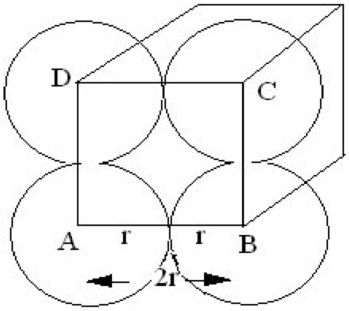
5. x:y:z=4:1:2
6. Reciprocals $\frac{1}{4}:\frac{1}{1}:\frac{1}{2}$
7. LCM=4
8. $4\times\frac{1}{4}: 4\times1 : 4\times\frac{1}{2}$

The miller indices are (hkl) = (1 4 2)

5). Find the atomic packing factor of SC, FCC and BCC structures. (Dec2014)

(1).Simple cubic structure (SC):

There is only one lattice point at each of the eight corners of the unit cell. Since each atom in the corner is shared by 8 unit cells,therefore the total number of atoms in one unit cell is (1/8) ×8 = 1



The nearest neighbor distance 2r = a

The number of lattice points per unit cell, n = 1

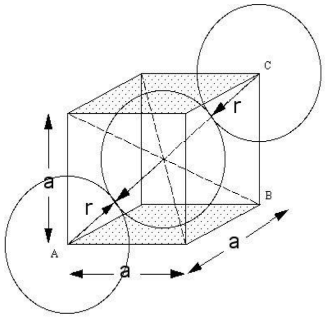
Volume of an atom in the unit cell is $v = \frac{4}{3}\pi r^3$

Volume of unit cell = $V = a^3 = (2r)^3$

Packing factor is $P.F = \frac{n\ v}{V} = \frac{1\times \qquad {}^{3}}{3\times 8r^{3}} = \frac{\Pi}{6} = 0.52 = 52\%$

(2).Body centered cubic structure (BCC):

In a BCC structure eight atoms are present at eight corners and one atom at the body center. Therefore the number of atoms per unit cell is, n = [(1/8)×8]+1 = 2



The lattice constant is

$(AB)^2 = a^2 + a^2 = 2a^2$

$(AC)^2 = (AB)^2 + (BC)^2$

$(4r)^2 = 2a^2 + a^2 = 3a^2$

Lattice constant $a = \frac{4r}{\sqrt{3}}$

Volume of an atom in the unit cell is v = $\left(\frac{4}{3}\Pi r^3\right)$

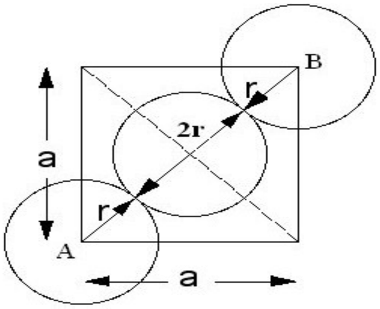
Volume of the unit cell V = a^3 = $\frac{64r^3}{3\sqrt{3}}$

Atomic packing factor = $\frac{n\,v}{V} = \frac{2\times 4\Pi r^3\times 3\sqrt{3}}{3\times 64r^3} = \frac{\sqrt{3}\,\Pi}{8} = 0.68 = 68\%$

(3).Face centered cubic structure (FCC):

In a FCC structure eight atoms are at the corners of the unit cell and six atoms are present at the center of the six faces. Each corner atom is shared by 8 unit cells and the face centered atom is shared by 2 surrounding unit cells.

∴ The number of atoms per unit cell is, n = $\frac{1}{8}\times 8 + 6\times\frac{1}{2} = 4$



In the diagram,

AB = 4r

$(AB)^2 = a^2+a^2$

$(4r)^2 = 2a^2$

Lattice Constant a = $\frac{4r}{\sqrt{2}} = 2\sqrt{2}r$

Nearest neighbor distance 2r = $\frac{a\sqrt{2}}{2}$

Volume of an atom in the unit cell is ,v = $\left(\frac{4}{3}\Pi r^3\right)$

Volume of unit cell, V = a^3 = $\frac{64r^3}{2\sqrt{2}}$

∴ The packing factor = $\frac{n\,v}{V} = \frac{4\times 4\Pi r^3\times 2\sqrt{2}}{3\times 64r^3} = \frac{\sqrt{2}\,\Pi}{6} = \frac{\Pi}{3\sqrt{2}} = 0.74 = 74\%$

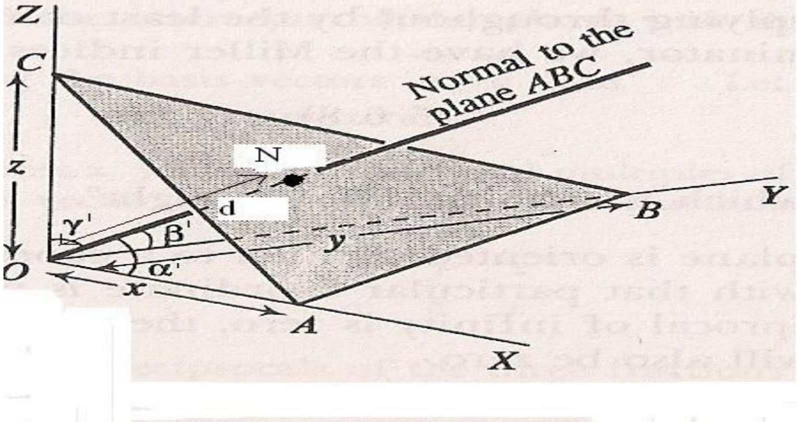
6). Determine the interplanar spacing for (110) planes for copper which has FCC structure and atomic radius 0.127nm. (Dec2014)

Lattice constant, a = $2\sqrt{2}r = 2\sqrt{2}\times 0.127\times 10^{-10}$ = 0.36nm

The interplanar spacing , $d = \frac{a}{\sqrt{(h^2+k^2+l^2)}} = \frac{0.36\times 10^{-10}}{\sqrt{(1^2+1^2+0)}}$ = 0.256nm

6). Obtain an expression for the interplanar distance in cubic crystal in terms of miller indices. (Dec2014)

Consider a plane ABC which belongs to a family of planes represented by the Miller indices (h k l). The perpendicular ON from the origin O to the plane represents the inter-planar spacing d=ON of this family of planes. Let ON make an angle α^1, β^1, γ^1 with the x, y, z axes respectively.



INTERPLANAR SPACING

The intercepts of the plane on the three axes are

OA= a/h ; OB= a/k ; OC=a/l

Where 'a' is the length of the cube edge.

Then from figure, we have

$$\cos A^{1} = \frac{d}{OA} = \frac{d}{a/h} = \frac{dh}{a}$$

$$\cos B^{1} = \frac{d}{OB} = \frac{d}{a/k} = \frac{dk}{a}$$

$$\cos \Gamma^{1} = \frac{d}{OC} = \frac{d}{a/l} = \frac{dl}{a}$$

But, $\left[\cos^2\left(A^1\right)+\cos^2\left(B^1\right)+\cos^2\left(\Gamma^1\right)\right]=1$ for the orthogonal coordinates

Substituting the values of $\cos A^1$, $\cos B^1$, $\cos \Gamma^1$ in the above equation

We get, $\left(\frac{d}{OA}\right)^2+\left(\frac{d}{OB}\right)^2+\left(\frac{d}{OC}\right)^2=1$

$$\left(\frac{dh}{a}\right)^2+\left(\frac{dk}{b}\right)^2+\left(\frac{dl}{c}\right)^2=1$$

$$d^2\left[\left(\frac{h}{a}\right)^2+\left(\frac{k}{b}\right)^2+\left(\frac{l}{c}\right)^2\right]=1$$

$$d_{(hkl)} = \frac{1}{\sqrt{\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}\right)}}$$

For a cubic structure, a=b=c.

Therefore, for a cubic structure $d_{(hkl)} = \frac{a}{\sqrt{\left(h^2+k^2+l^2\right)}}$

This is the expression for the interplanar distance 'd' in cubic crystal in terms of miller indices.

7). Sketch and explain the structure of diamond crystal. (Dec2014)

Crystal structure of Diamond:

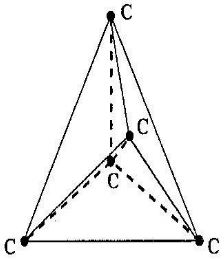
The diamond lattice can be considered as the superposition of 2 fcc sublattices one of which is displaced from the other along the body diagonal of the unit cell by $(1/4)^{th}$ the length of the diagonal. Thus a carbon atom will be present at the center of tetrahedron, with four carbon atoms of the other sublattice as its nearest neighbors located at four corners of the same tetrahedron. Thus the co-ordination number(N) of diamond is 4 and there will be 8 carbon atoms present per unit cell(n)**.** That is each carbon atom is at the center of a tetrahedron,4 carbon atoms are at diagonally opposite in the planes. The lattice constant is $3.5A^o$ and the bond length is $6.56A^o$.

The semiconductors like Si and Ge are having diamond structure.

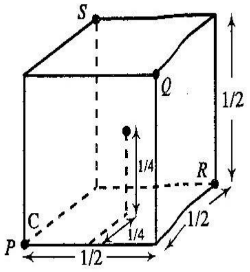
An FCC lattice has 4 atoms/unit cell. But the diamond lattice comprises of two interpenetrating FCC sub-lattice.

Therefore, Number of atoms per unit cell in the diamond lattice is 2x4 = 8.

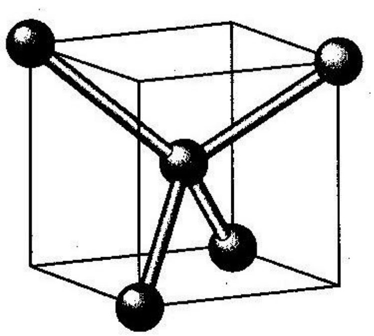
And Atomic Packing Factor = $\frac{8}{a^3}\times\frac{4}{3}\Pi r^3 = \frac{8}{\left(\frac{8}{\sqrt{3}}r\right)^3}\times\frac{4}{3}\Pi r^3 = \frac{\sqrt{3}}{16}\Pi = 34\%$



TETRAHEDRON FORMATION BY C ATOMS



CO-ORDINATES OF C ATOMS

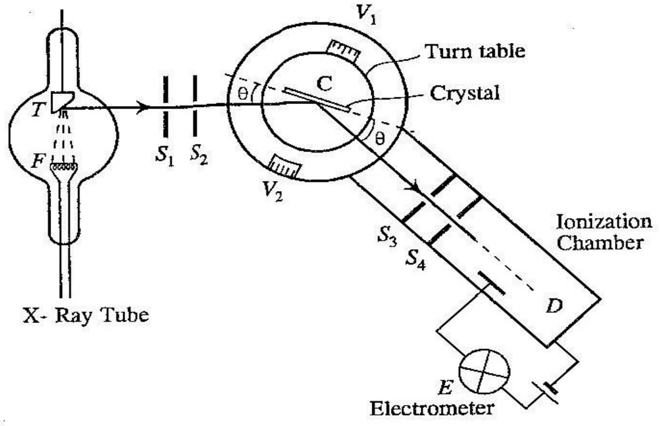


Type equation here.

Considering a corner of the cube and expressing the coordinates in terms of the edge length of the cube, the position of each of the C atoms can be evaluated.
The coordinates of the central C atoms are, (¼ , ¼, ¼).
The coordinates of the central C atoms at the positions PQR and S are respectively,
000 , $\frac{1}{2}0\frac{1}{2}$, $\frac{1}{2}\frac{1}{2}0$, $0\frac{1}{2}\frac{1}{2}$.

8). Explain how bragg's law verified using Braggs X-ray spectrometer. (Dec2014)

The schematic diagram of Bragg's x-ray spectrometer is shown in fig. It is used to determine lattice constant and inter-planar distance'd'. It has 1) x-ray source 2) A Crystal fixed on a circular table provided with scale and vernier. 3) Ionization chamber.

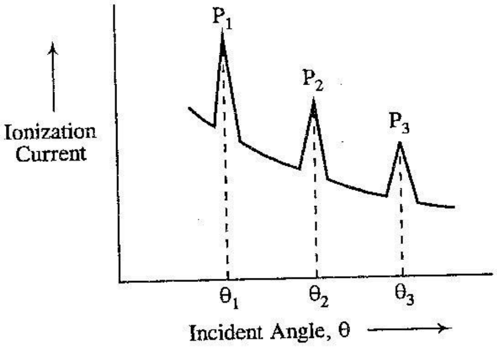


BRAGG'S X–RAY SPECTROMETER

A collimated beam of x-rays after passing the slits S_1 and S_2 is allowed to fall on a crystal C mounted on a circular table. The table can be rotated about vertical axis. Its position can be measured by vernier V_1. An ionization chamber is fixed to the longer arm attached to the table. The position of which is measured by vernier V_2. An electrometer is connected to the ionization chamber to measure the ionization current produced by diffracted x-rays from the crystal. S_3 and S_4 are the lead slits to limit the width of the diffracted beam. Here we can measure the intensity of the diffracted beam.

If x-rays incident at an angle 'θ' on the crystal, then reflected beam makes an angle 2θ with the incident beam. Hence the ionization chamber can be adjusted to get the reflected beam till the ionization current becomes maximum.

A plot of ionization current for different incident angles to study the x-ray diffraction spectrum is shown in fig.



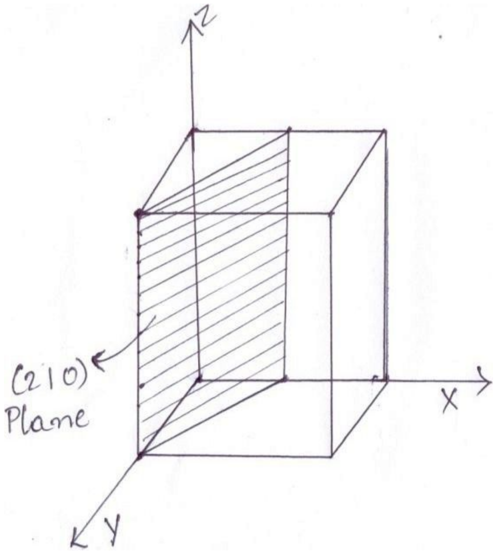
VARIATION OF IONIZATION CURRENT

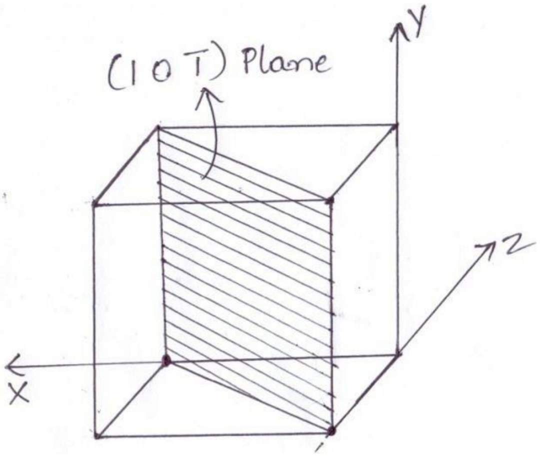
The rise in Ionization current for different values of ‘θ’ shows that Bragg’s law is satisfied for various values of ‘n’. i.e. $2d\sin\theta = \lambda$ or 2λ or 3λ etc. Peaks are observed at θ_1, θ_2, θ_3 etc. with intensities of P_1, P_2,P_3 etc.

i.e. $2d\sin\theta_1 : 2d\sin\theta_2 : 2d\sin\theta_3 = \lambda : 2\lambda : 3\lambda$

The crystal inter-planer spacing‘d’ can be measured using $2d\sin\theta = n\lambda$

9)Draw the crystal planes (210) and (10$\bar{1}$) in a cubic crystal. (Dec2014)





10) Mention the geometrical configuration of the seven crystal systems. (June2015)

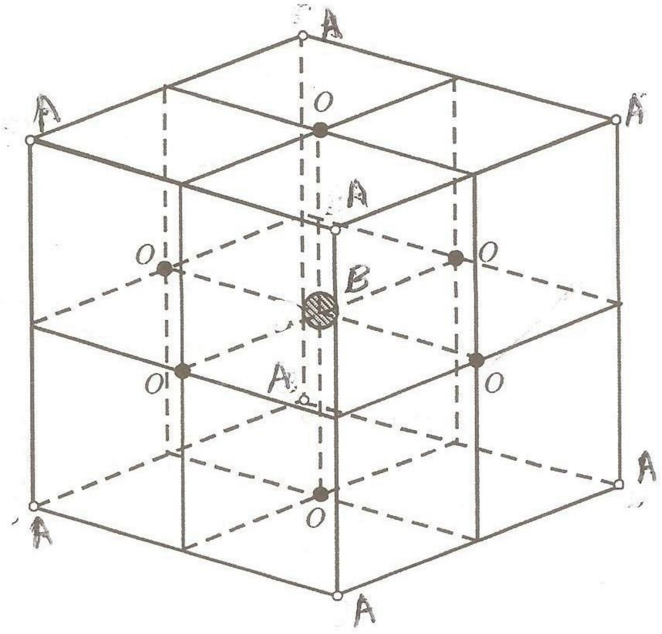
CRYSTAL SYSTEMS

Different crystalline solids have different crystal structures. A type of crystal structure is defined by six crystal parameters a, b, c, α, β and γ. At the first glance, it appears that there is a large number of combinations of these parameters leading to a large number crystal types. But, it is restricted by the condition that while filling the lattice with unit cells, there should be no void. Actually, there are only 7 such combinations leading 7 crystal systems. Further, Bravais mathematically showed that within the 7 crystal systems, 14 sub-arrangements (14-Bravais lattices) are possible in Bravais lattice.

Sl. no.	**Crystal System**	**Basis vectors**	**Interfacial Angles**	**Examples**	**Bravais lattices**
1	Cube	$a = b = c$	$\alpha = \beta = \gamma = 90^o$	Iron, Copper, aluminum, silicon, lead..	1. SC 2. BCC 3. FCC
2	Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90^o$	Tin, Barium titanate..	1.Simple 2.Body centered
3	Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^o$	Uranium, Sulphur, Sodium Sulphate..	1.Simple, 2.face centered, 3.body centered 4.base centered
4	Monoclinic	$a \neq b \neq c$	$\alpha = \gamma = 90^o \neq \beta$	Phosphorous, Tin Fluoride	1.Simple and 2.base centered
5	Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^o$	Potassium dichromate.	1.Simple triclinic
6	Rhombohedral (trigonal)	$a = b = c$	$\alpha = \beta = \gamma \neq 90^o$	Mercury, Bismuth, Antimony..	1.Simple trigonal
7	Hexagonal	$a = b \neq c$	$\alpha = \beta = 90^o$, $\gamma = 120^o$	Zinc, Magnisium, Cadmium..	1.Simple hexagonal

11). Sketch and describe the perovskite structure. (June2015)

PEROVSKITE STRUCTURE



CRYSTAL STRUCTURE OF PEROVSKITE

*The structure of calcium titanium oxide ($CaTiO_3$) is called Perovskite structure.

*The general chemical formula for Perovskite compounds or oxides is ABO_3 (or ABX) where A and B are two cations (+ve ions) or metal ions, and O_3 (or X) is an anion that bonds with the former two.

*A and B are of very different sizes, A being larger than B.

*Some of the other perovskites are $BaTiO_3$, $SrTiO_3$, $PbTiO_3$ and$MgSiO_3$.

*Perovskites have non-bravais cubic lattice.

*In a cubic unit cell A atoms occupy cube corner positions, B the body center position and Oxygen atoms the face center positions.

*Hence the number of A type atoms / unit cell = 8 × 1/8 = 1

*Number of B type atoms / unit cell =1

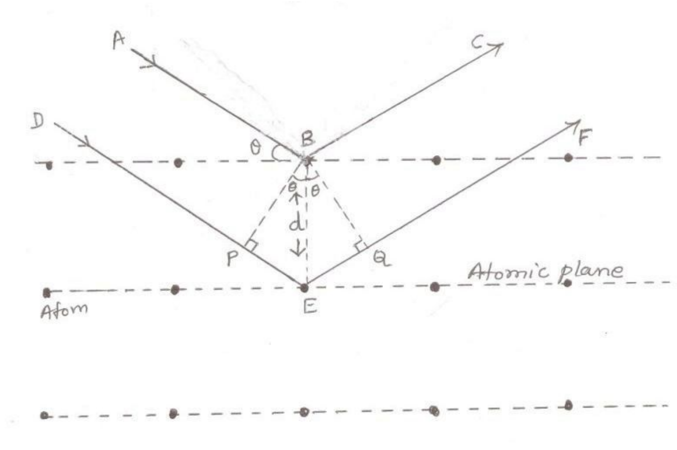
*Number of oxygen atoms / unit cell = 6 × ½ = 3

*Hence the number of ABO_3 molecules / unit cell =1

*An alternate arrangement is also possible for perovskites. In this case, an A atom is at the body center, B atoms are at the corners and O atoms are at the centre of the edges. The number of atoms of each type remains the same

12). Derive Bragg's equation. (June2015)

BRAGG'S LAW



Consider a monochromatic beam of x-rays incident on a set of parallel crystal planes, at a glancing angle θ.

The ray AB, incident on the atom at B is reflected along BC.

The ray DE incident on the atom at E is reflected along EF, parallel to BC.

Draw BP perpendicular to BC and BQ to EF.

The path difference between the reflected rays BC and EF is $\delta = PE + EQ$

From ΔBPE, $\sin\theta = \frac{PE}{BE} = \frac{PE}{d}$ or $PE = d\sin\theta$, d being the inter-planar distance.

Similarly, $EQ = d\sin\theta$

Therefore $\delta = d\sin\theta + d\sin\theta = 2d\sin\theta$

Bragg's reflections give rise to constructive interference if $\delta = n\lambda$ where $n = 1,2,3,$.

Hence the condition for constructive interference is $2d\sin\theta = n\lambda$

This equation is known as Bragg's law.

13). The atomic radius of gold is 0.144nm. Determine the interplanar distance for (110) planes assuming that gold belongs to FCC system. (June2015)

w.k.t

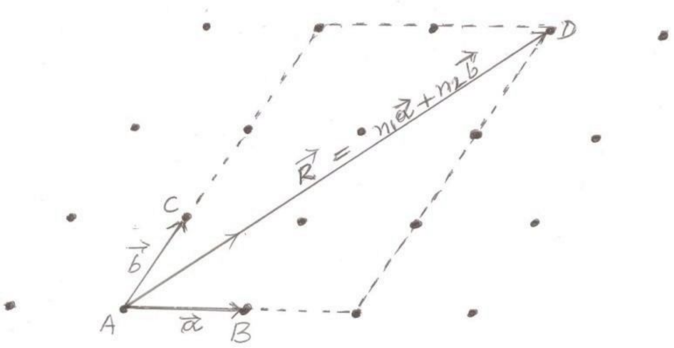
$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$d = \frac{2\sqrt{2r}}{\sqrt{h^2 + k^2 + l^2}}$$

$$d = \frac{2*1.414*0.144*10^{-9}}{\sqrt{1^2+1^2+0^2}}$$

14). With the help of vector diagram explain the terms basis vectors, lattice vector, interfacial angles and crystal parameters of a space lattice.(June2015)

Basis vectors and Lattice vector:



Consider a two -dimensional crystal lattice (a plane of atoms).

To specify a lattice point in space lattice, a coordinate system is necessary.

Consider a coordinate system with origin at some lattice point A.

Consider two nearest, non-collinear lattice points B and C.

Draw the vectors $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BC} = \vec{b}$.

Consider a lattice point D and draw the position vector $\overrightarrow{AD} = \vec{R}$.

Then $\vec{R} = n_1\vec{a} + n_2\vec{b}$ where n_1 and n_2 are integers (for 2D).

For a three -D lattice, we can write $\vec{R} = n_1\vec{a} + n_2\vec{b} + n_3\vec{c}$

where $\vec{a}\,\vec{b}$ and $\vec{c}$ are the basis vectors / unit vectors / translation vectors /primitive vectors and $\vec{R}$ is the lattice vector.

Basis vectors are the smallest vectors drawn along three repetitive directions of atoms, using which position of any lattice point may be expressed.

A position vector expressed in terms of basis vectors is called lattice vector.

The direction of a basis vector is a crystal axis. There are three crystal axes w.r.t., a given origin.

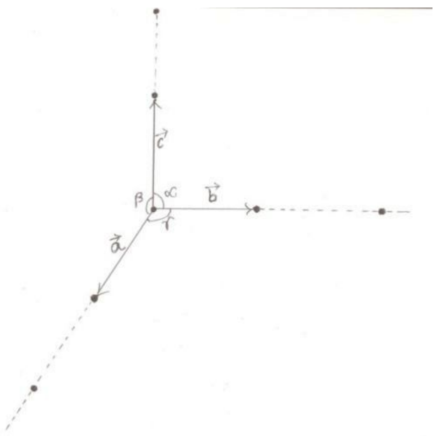
The direction of a lattice vector is a crystal direction. There are many crystal directions w.r.t., a given origin.

Interfacial angles:

The angles between the basis vectors are called the interfacial angles.

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the basis vectors, joining the origin to the nearest non-collinear lattice points.

Let α, β and γ be the minimum angles between the vectors $\vec{b}\,\&\,\vec{c}$, $\vec{a}\,\&\,\vec{c}$, and $\vec{a}\,\&\,\vec{b}$ respectively. Then α, β and γ are the interfacial angles.



Lattice parameters:

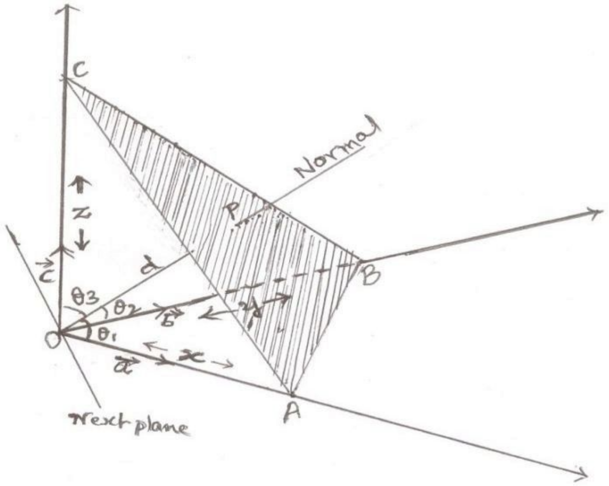
The type of space lattice or the type of a crystal structure is determined by a set of three basis vectors and three interfacial angles. These six parameters are called crystal parameters.

Or

Three basis vectors and three interfacial angles specify a unit cell completely and are called the crystal parameters.

15) Derive an expression for interplanar distance in terms of Miller indices.

(June2015)



Consider a coordinate system with O as the origin in a crystal lattice.

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the basis vectors along the three crystal axes taken in order.
Let ABC be one of the parallel planes represented by the Miller indices (h k l).
Let its intercepts with the crystal axes be x, y&z in the same order.
Let the next parallel plane pass through the origin itself.
Draw OP normal to ABC. Then OP = d is the inter-planar distance.

Let the angles made by OP with the crystal axes be θ_1, θ_2 and θ_3, in the same order.

Then OP becomes a common adjacent side to the three right angled triangles OPA, OPB and OPC.

Then OA=*x*, OB=*y*& OC=*z* will be the corresponding hypotenuses.

From the definition of Miller Indices,

$$(h\ k\ l) = \left(\frac{a}{x}\ \frac{b}{y}\ \frac{c}{z}\right)$$

Then $(x\ y\ z) = \left(\frac{a}{h}\ \frac{b}{k}\ \frac{c}{l}\right)$

From ΔOPA,

$$\cos\Theta_1 = \frac{OP}{OA} = \frac{d}{x}$$

But $x = \frac{a}{h}$

$$\therefore \cos\Theta_1 = d\,\frac{h}{a}$$

Similarly,

$\cos\Theta_2 = d\,\frac{k}{b}$ and $\cos\Theta_3 = d\,\frac{l}{c}$

By squaring and adding cosine terms we get

$$\cos^2\Theta_1 + \cos^2\Theta_2 + \cos^2\Theta_3 = d^2\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)$$

From the condition of orthogonality,

$$\cos^2\Theta_1 + \cos^2\Theta_2 + \cos^2\Theta_3 = 1$$

$$\therefore d^2 = \frac{1}{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}$$

OR

$$d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}$$

For cubic system, $a = b = c$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

16) Define coordination number and packing factor. Compute the packing factor for BCC crystals. (June2015)

COORDINATION NUMBER

In a crystal all the atoms have an identical surrounding.

The number of equidistant nearest atoms surrounding an atom in a crystal is called the coordination number of the crystal.

CN of SC is 6, BCC is 8 and FCC is 12.

ATOMIC PACKING FACTOR (PF)

Packing factor is the ratio of volume occupied by all the atoms in the unit cell to the total volume of the unit cell.

For a cubic structure, volume of the cell = a^3 where a is the lattice constant.

Volume of the atom = $\frac{4}{3}\Pi R^3$ where R is the atomic radius.

If there are n atoms/unit cell, the occupied volume = $n\frac{4}{3}\Pi R$

Therefore, PF = occupied vol. / total vol. = $\dfrac{n\frac{4}{3}\Pi R}{a^3}$

Or $\qquad$ $\text{PF} = \frac{n}{a^3} \times \frac{4}{3}\Pi R$

BCC: In this case, $n = 2$ and $a = \frac{4}{\sqrt{3}}R$

$$\text{PF} = \frac{2}{\left(\frac{4}{\sqrt{3}}R\right)^3} \times \frac{4}{3}\Pi R^3 = \frac{\sqrt{3}}{8}\Pi = 0.68$$

17) In a calcite crystal, second order Bragg's reflections occur from the planes with d-spacing 3Å, at a glancing angle of 24°. Calculate the path difference between X-rays reflected from the two adjacent planes. Also, Calculate the wavelength of the X-rays. (June2015)

$2\, d \sin\theta = n\lambda$

Therfore:

$\text{P.D} = 2\, d \sin\theta$

$\text{P.D} = 2*3*10^{-10}*\sin 24$

$\text{P.D} = 2.44*10^{-10}\text{m}$

And $\Lambda = 1.22*10^{-10}\text{m}$

Module -5

Shock waves and Science of nanomaterials

1). What are nano materials? Explain carbon nano tubes and their physical properties. Mention few applications of carbon nano tubes. (June2013) (jan2011)

Matter arranged by exercising control over lengths of one to hundred nano meter and the formulating structures exhibit characteristics that are specific to their size and dimensions, the resulting materials are called nano materials. Inorganic and organic nano materials are the two types. Carbon Nanotubes is a molecular tubes of about 10,000 times thinner than a human hair, consist of rolled up sheets of carbon hexagons as shown in the figure. Carbon nanotubes have the potential for use as minute wires, medical matrix for growth of bone cells, battery storage, Field Emission Displays and much more. These nanotubes are single molecules measuring a few nanometers in diameter and several microns in length.

The nano tubes conducting properties depend upon how the two ends of the sheet meet along. For a particular way of arrangement of atoms along the meeting line makes the entire tube conducting like a metal and for another type of arrangement, the tube behaves like a semi conductor .When one type nano tube is rolled inside another type we get multi walled nano type. Using semiconductor nano tubes transistors and logic gates are constructed. Nano wires are made using nano tubes. carbon nanotubes exist as a macro-molecule of carbon, analogous to a sheet of graphite rolled into a cylinder. However, when rolled, the carbon arrangement becomes very strong. In fact, nanotubes have been known to be up to one hundred times as strong as steel and almost two millimeters long. These nanotubes have a hemispherical "cap" at each end of the cylinder.

2). write a note on carbon nano tube. Discuss various quantum structures. (June2013) (July2011)

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Quantum wells

A quantum well is a potential well that confines particles, which were originally free to move in three dimensions, to two dimensions, forcing them to occupy a planar region. The effects of quantum confinement take place when the quantum well thickness becomes comparable at the de Broglie wavelength of the carriers (generally electrons and holes) leading to energy levels called "energy subbands", i.e., the carriers can only have discrete energy values.

The reduced amount of active material in quantum wells, leads to better performance in optical devices such as laser diodes. By doping either the well itself, or preferably, the barrier of a quantum well with donor impurities, a two-dimensional electron gas (2DEG) forms. Such as structure forms the conducting channel and has interesting properties at low temperature.

Quantum wires

In condensed matter physics, a quantum wire is an electrically conducting wire, in which quantum effects are affecting transport properties. Due to the quantum confinement of conduction electrons in the transverse direction of the wire, their transverse energy is quantized into a series of discrete values E_0 ("ground state" energy, with lower value), E_1,... . In quantum wires an exact calculation of the transverse energies of the confined electrons has to be performed to calculate a wire's resistance. Following from the quantization of electron energy, the resistance is also found to be quantized. The importance of the quantization is inversely proportional to the diameter of the nanowire for a given material. From material to material, it is dependent on the electronic properties, especially on the effective mass of the electrons. For metals, quantization corresponding to the lowest energy states is only observed for atomic wires. Their corresponding wavelength being thus extremely small they have a very large energy separation which makes resistance quantization perfectly observable at room temperature.

Quantum dots

A quantum dot is a semiconductor whose excitons are confined in all three spatial dimensions. As a result, they have properties that are between those of bulk semiconductors and those of discrete molecules. Researchers have studied quantum dots of transistors, solar cells, LEDs, and diode lasers. They have also investigated quantum dots as agents for medical imaging and hope to use them as qubits. Quantum dots are semiconductors whose conducting characteristics are closely relate

3). Describe with simple illustrations, the two methods of preparation of nanomaterials. (July2013)

The nanomaterials are manufactured in two approaches.

1)Toptobottomapproach:inthismethodthemacroormicrosizeparticlesaremilledtill weget the nanosize particles.

2)Bottomtotopapproach:Inbottomtotopapproach,thevapourizedstateof atoms/molecules are condensed or arranged till we get the nanosize particles.

The different synthesis methods are

1)Arc Method

2)Laser Method

3)Chemical Vapor Deposition:

4)Ball Millingand

5)Sol-gel synthesis

Arc-dischargeandlaservaporizationarecurrentlytheprincipalmethodsforobtainingsmall quantitiesofhighqualitynanomaterialsspeciallyCNTs.However,bothmethodssufferfrom drawbacks.Thefirstisthatbothmethodsinvolveevaporatingthecarbonsource,soithas beenunclearhowtoscaleupproductiontotheindustriallevelusingtheseapproaches.The secondissuerelatestothefactthatvaporizationmethodsgrownanomaterialsinhighly tangledforms,mixedwithunwantedformsofcarbonand/ormetalspecies.TheCNTsthus producedaredifficulttopurify,manipulate,andassembleforbuildingnanotube-device architecturesforpracticalapplications.Onlysol-gelsynthesiscanproducematerials(both metals and ceramics) at low temperatures shows the following advantages

- Large quantities (to be commercially viable) relatively cheap,
- Synthesize almost any material,
- Co-synthesize two or more materials simultaneously,
- Coatoneormorematerialsontoothermaterials(metalorceramicparticulates, and three- dimensional objects),
- Produce extremely homogeneous alloys and composites,
- Synthesize ultra-high purity (99.9999%) materials,
- Tailor the composition very accurately even in the early stages of the process, because the synthesis is actually performed on an atomic level,
- Precisely control the microstructure of the final products, and precisely control the physical, mechanical, and chemical properties of the final products.

4). Explainquantum structures.(July2013)

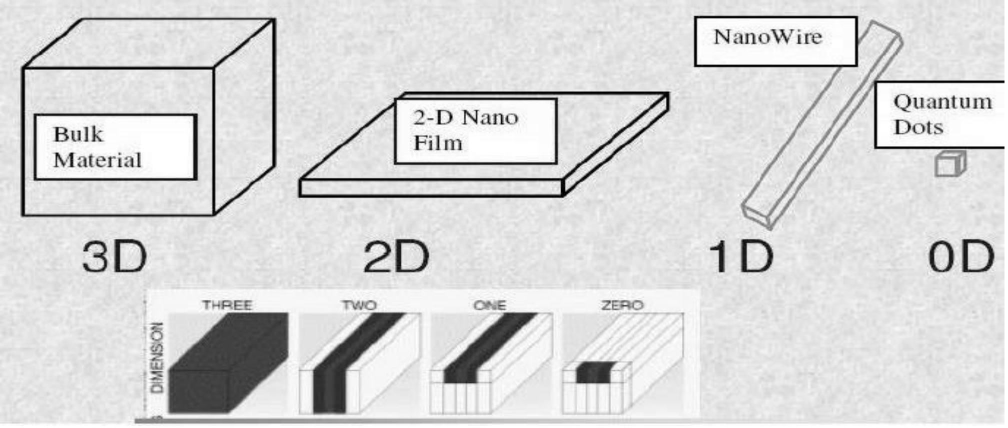
Therearefourtypesofquantumstructuresvaryinthesizerangeof1nmto100nmLargecluster s/Particles(3-D),Thinfilmsandlayers(2-D),RodsandWires(1-D),SmallClusters: Quantum Dots (0-D).

Nanomaterials of lower dimensions in the nanoscale are as follows:-

Bulkmaterialof3-dimensionsasareductioninonedimensiontheresultingstructurein2dimensions is called a film.

Ifthereductionisin2dimensionstheresultingstructureisin1dimensionitis quantumwire.Ifthereductionisinallthe3directionstheresultingstructureisquantumdot. Itisalsocallednanoparticleorcluster.Chargecarriersareabletomoveinalldirectionsina 3-D material,confinedtoaplaneinafilminonlyonedirectionina quantumwirebutina0-D structure they will remain confined to a very small space.

Thefilm,wireanddotshavecertainthicknessforthematerialalongthedirection wherewe saythecorrespondingdimensionisabsent.Thethicknessislessthanthemeanfree pathfortheelectroninthematerial;thisthicknesswillbeinnanometerrange.Thematerial along these directions exhibits mesoscopic properties.



5) What are shock waves ? Explain the experimental method of producing shock waves and measuring its mach number using Reddy shock tube. (Dec2014)

Any flood that propagates at supersonic speeds, gives rise to a shock wave. They are characterized by sudden increase in pressure and density of gas through which it propagates.

Experimental analysis using Reddy tube :

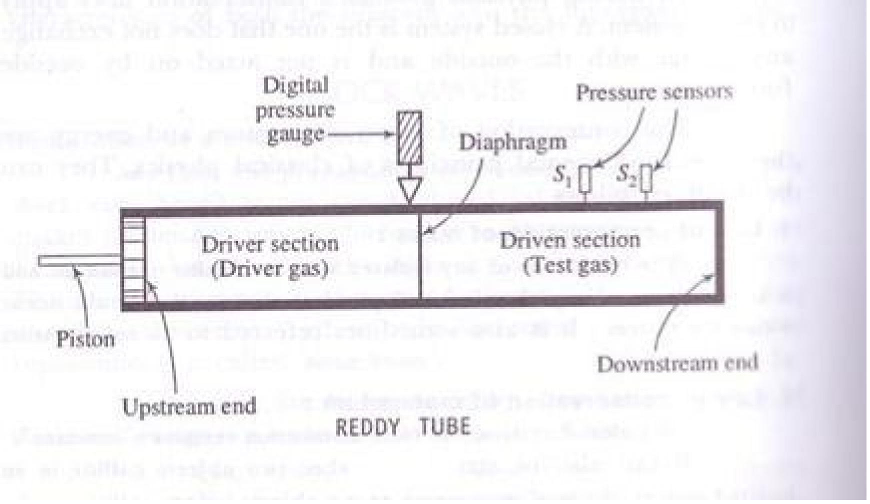
Aim: 1.To determine the speed of the primary shock wave and determine the Mach number.

2. To determine the $\frac{p_2}{p_1}$across the primary shock wave and evaluate p_5&T_5across the reflected shock wave and compare with the calculated values.

Apparatus required: Reddy tube, digital pressure gauges ,2 pressure sensors, oscilloscope, diaphragm, blanking plate ,vacuum pump.

Reddy shock tube :

Reddy shock tube consist is hand operated shock tube capable of producing shockwaves by using human energy. Its long cylindrical tube with two sections areseparated by diaphragm. It's one end is fitted with piston and other end is closed or open to the surroundings.



Construction:

Reddy shock tube consists of cylindrical stainless steel tube of about 30mm diameter and length 1m .It is divided into two section each of length 50cm ,one is driver tube and other one is driven tube separated by 0.1mm thick of diaphragm.

1. The Reddy tube has a piston fitted at far end of the driver section and far end of the driver section is closed.
2. Digital pressure gauge is mounted in the driver section next to the diaphragm .Two piezoelectric sensors s_1 and s_2 are mounted 70mm apart towards the closed end of the shock tube . A port is provided at the closed end of the driven section for the filling the test gas to the required pressure.

3. The driver section is filled with a gas termed as driver gas ,which is held at relatively high pressure due to compressing action of the piston .the gas driven section is termed as driven gas.

Working:

The driver gas is compressed by pushing the piston hard into driver tube until diaphragm ruptures .Then driver gas rushes into the driven section and pushes driven gas towards the far downstream end. This generate moving shock wave instantaneously raises the temperature and pressure of the driven gas as the shock move over it .

The propagating primary shock waves are reflected from the downstream end. After the reflection, the test gas undergoes further compression which boosts it temperature and pressure to still higher values by reflected shock waves.

This state of high values of pressure and temperature is sustained or continuous (it takes time of the order milliseconds) at the downstream end until an expansion wave reflected from the upstream end of the driver tube arrives there and neutralizes compression partially .these expansion waves are created at the instant the diaphragm is ruptured and they travel in the direction of shock waves.

Actual creation of shock waves are is depends on the properties of driver and test gases and dimension of the shock tube .pressure rise caused by primary shock waves and reflected shock waves are sensed by sensors s_1 and s_2 respectively and they are recorded in digital cathode ray(CRO).the pressure sensors are piezoelectric transducers.

Since the experiment involves 1millisecond duration measurement, rise time of oscilloscope should be few microseconds with bandwidth of 1 MHz or more is required. From the recording the CRO, shock wave arrival time is calculated and also Mach number, pressure and temperature can be calculated.

Procedure:

1. The end of the driven tube is closed with a blanking plate .The pressure inside the driven section is reduced by connecting it to a vacuum pump, and valve is then closed.

2. The diaphragm is fitted into the reddy tube. the pressure sensors are connected to the CRO via the coupler & CRO is set for observations.

3. The piston is driven hard into the driver tube till the rupture sound of the diaphragm is heard. The pressure gauge reading in driver tube is noted.

4. To signals of pressure rise are seen in the CRO display, the first one appears when primary shock passes the first sensor and the second one, when it passes the second sensor. From the CRO display, the time taken 't' for the shock to travel between the two pressure sensors is measured. The distance x between two pressure sensors is measured with graduated scale.

5. The experiment is repeated 3 or 4 times, and the average value is taken and calculated the following.

Calculations:

1. Evaluation of Mach number, M:

Shock speed of primary shock wave, Us=$\frac{x}{t}$, Mach number, M=$\frac{U_s}{a}$

6) Give the graphical representation of density of states with equation for 0D,1D,2D and 3D structures. (Dec2014)

Density of states (Dos) for various quantum structures:

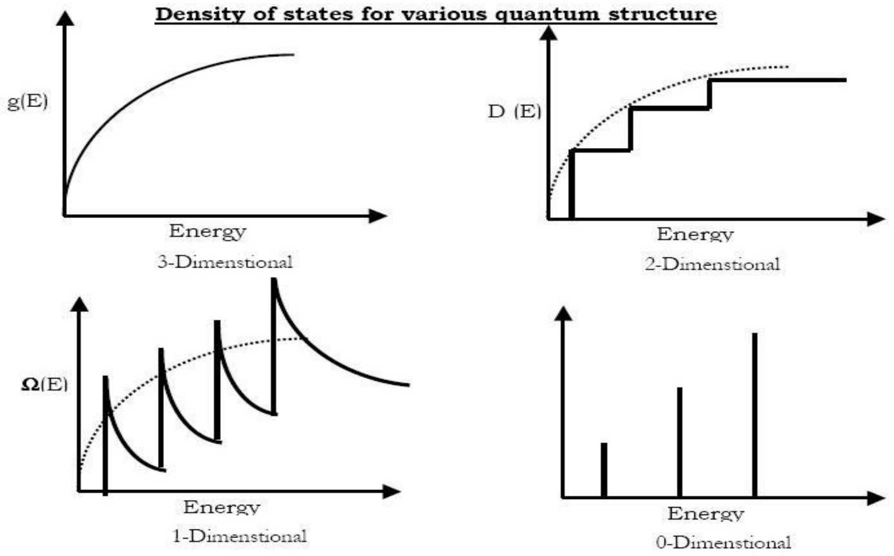
Treating the free electrons as electron gas in one, two & three dimensions, we can arrive at expression for density of states for the respective cases.

The density of states (Dos) for the various quantum structures as function of energy are shown in fig.

The DOS denoted as g(E) for 3D materials, various continuously. The density will be typically of the order of the order of $10^{26}/m^3$ev. for a 2 -D structure,

The DOS subbands in a 2-D structure is given by , D(E)=$\frac{m^{*}}{\pi\hbar^2}$

the dos denoted as D (E) varies as a step-function i.e., there will be sudden rise in D (E) at certain energy values such as $E_1 E_2$,,, (fig .2b).it happens because the energy progress (in terms of energy states) of the first sub band continues even when the second subband starts.



Density of states for various quantum structure

So. In the second subband, apart from to its own energy states, there will be additional contribution from the continuation of energy states of first subband. Hence the dos rises at once at the beginning of second subband .hence DOS rises at once at the beginning of second subband. Again, the combined the energy progress of first & second subbands enter into the third subband ,& DOS rises further at the commencement of third subband. Thus the overall variation takes a stair case shape. however, the locus of all the corners of the will be a parabola. If the thickness of the 2-D material is increased, more subbands will be created which results increase of the step width. If thickness increase continues, then it is limit ,the material reaches 3- dimensions at which time, the innumerable steps would be seen merged into an envelope of parabolic variation for DOS (as it should be for a 3-D material).the density is typically of the order of$10^{18}/m^2$eV for 2-D structure.)

D (E) dE = $\frac{m^*}{\hbar^2} \sum H(E - E_i)$ dE, Where m* is the effective mass of electron in the structure, and H (E-E_i) is a step function called Heaviside function.its values are zero for E <E_i and, 1 for E ≥ E_i

E_i is the i^{th}energy level in the subband. For a 1- D structure, i.e., for a Quantum wire, the DOS variation is not smooth as in the case of 3-D structure not even constant over the sub band also. The density hits peaks at energy values eE_1, E_2…, and decreases rapidly in the range in between

The DOS in this case is given by, electrons are confined in one or more directions by reducing the dimensions of the material in those directions, the density of the state's changes due to quantization if energy,

Which is a material reduced in 1-dimention to nano scale, there is Quantization of energy due to confinement of electrons in one direction the density of states for a quantum well o r2-D film is given by

$$\Omega(E)\, dE = \frac{\sqrt{2m}}{\Pi h} \sum \frac{n_i\, H(E - E_i)}{\sqrt{(E - E_i}}\, dE$$

Where H(H-E_i) is the Heaviside function.n_i is the degenerate function.

7) What are the properties of Carbon Nanotubes? (Dec2014)

Properties of CNT´S

Mechanical properties:

The tensile strength of CNT´s is much higher than steel or Kevlar and found to have highest tensile strength of all known materials so far. CNT´s can withstand larger strains than steel. They are highly elastic and bent without breaking. In addition to the weight of CNT´s is about only 1/6 of steel.

(2)Electrical properties:

They show electrical properties ranging from conductors to semiconductors depending on their structure, size and chirality. Their current carrying capacity is 1000 times that of copper conductivity of a CNT is maximum along its axis and very low in a perpendicular direction. hence they are equivalent to 1D conductors.

(3)Thermal properties:

Their thermal conductivity is more than twice that of diamond. they are very stable and retain their physical structure in vacuum even upto 2800 C.

(4)Chemical properties:

CNT's are chemically more inert compared to other forms of carbon.

8). What are the ultrasonic and supersonic waves? Describe in brief how the normal shock relationships are arrived. (Dec2014)

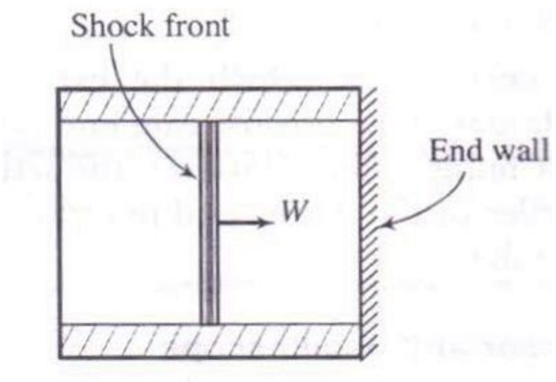
Ultrasonic waves: Ultrasonic waves are pressure waves having frequencies beyond 20.000 Hz. but they travel with the same speed as that of sound. Amplitude of the ultrasonic wave is also small.

Subsonic waves: If the speed of the mechanical wave or body moving in the fluid is lesser than that of sound then such a speed is referred to as subsonic and the wave is a subsonic wave. All subsonic waves have mach no<1.

<u>Rankine and Hugoniot equations (Shock relations):</u>

Consider a shock wave propagating with a speed W in a shock tube. The condition of the shock wave at the downstream end can be determined by solving the equations for conservation of mass momentum and energy as applied to the shock region .These equations are known as Rankine and Hugoniot relations.

Consider two regions with reference to the shock front, one which is ahead of shock front and other behind it .Both the regions are at far enough distances from the shock front so that equilibrium conditions attained in two regions where physical conditions such as pressure, density, temperature etc. are uniform.



PROPAGATING SHOCK WAVE

Let p_1, T_1, ρ_1 and ⍰$_1$ be the values of pressure ,temperature ,density and enthalpy before creation of the shock wave .Hence, these conditions attained in the region ahead of the shock wave.

Similarly,p_2, T_2, ρ_2 and ⍰$_2$ be the corresponding values after the diaphragm ruptures. Hence they are applicable in the region behind the wave. Then the three conservation relations which corresponds to the conservation laws are as stated in the following taking shock stationary frame of reference.

$$\rho_1 u_1 = \rho_2 u_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

(Conservation of mass)

Where, u_1→ velocity of the fluid ahead of the shock.

u_2→ Velocity of the fluid following the shock.

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

(Concerning conservation of momentum)

⍰$_1 + \frac{u_1^2}{2} =$ ⍰$_2 + \frac{u_2^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (4)

(Concerning conservation of energy)

These 3 equations along with the equation of state

i.e PV =RT ………………………….(5)

Using the above equations ,the following normal shock relations called Rankine –hugoniot equations can be derived in which p_5&T_5 represent pressure and temperature at the downstream end behind the reflected shock wave.

1. $\frac{p_2}{p_1} = [1 + \frac{2\gamma}{\gamma+1} (M^2 - 1)]$

2. $$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left[\frac{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}{1+\frac{(\gamma+1)}{\gamma-1}\frac{p_2}{p_1}}\right]$$

3. $$\frac{p_5}{p_2} = \frac{(3\gamma-1)\frac{p_2}{p_1}-(\gamma-1)}{(\quad)\frac{p_2}{p_1}}$$

4. $$\frac{T_5}{T_2} = \frac{p_5}{p_2}\left[\frac{\frac{\gamma+1}{\gamma-1} + \frac{p_5}{p_2}}{1+\frac{\gamma+1}{\gamma-1}\frac{p_5}{p_2}}\right]$$

These are Rankine –Hugoniot equations.

9) Define shockwaves. Mention its properties. (July2015)

Shock waves are strong pressure disturbances that are generated by explosion or by plane flying.

Or

“Shock waves is defined as a surface which manifests as a discontinuity in fluid medium in which it is propagating with supersonic speed”

Or

Shock waves are pressure pulses of microsecond duration with pressure of 35-125MPa followed by a tensile wave.

Properties:

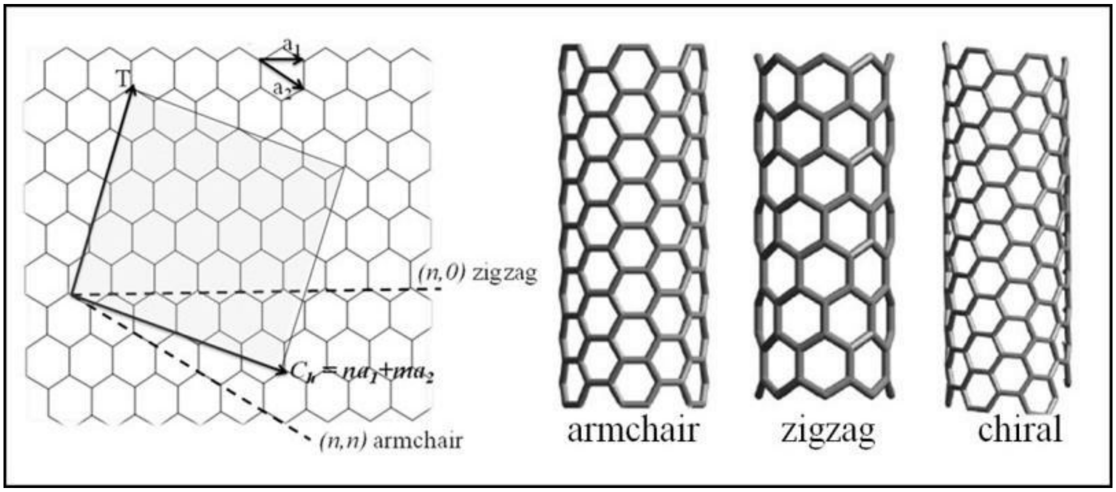
1. Shock waves travels through most media at a higher speed than an ordinary wave.
2. A shock waves arise at explosions, detonation, supersonic movements of bodies power full electric discharge.
3. When a shock waves possess through a matter it obeys laws of conservation mass, momentum & energy.
4. They are created in fluids.
5. Shock wave are characterized by an abrupt, discontinuous increase in pressure, temperature and density of the medium.

10).What are nanomaterial’s? Outline the structure of a carbon nano tube. (July2015)

Matter arranged by exercising control over lengths of 1 to 100 nano meter and the formulating structures exhibit characteristics that are specific to their size and dimensions, the resulting materials are called Nano materials.

Carbon Nanotubes is a molecular tubes of about 10,000 times thinner than a human hair, consist of rolled up sheets of carbon hexagons as shown in the figure. These nanotubes are single molecules measuring a few nanometers in diameter and several microns in length. The nano tubes conducting properties depend upon how the two ends of the sheet meet along. For a particular way of arrangement of atoms along the meeting line makes the entire tube conducting like a metal and for another type of arrangement, the tube behaves like a semiconductor. When one type nano tube is rolled inside another type we get multi walled nano type. Using semiconductor nano tubes transistors and logic gates are constructed. Nano wires are made using nano tubes.

Nano tubes non-reactive and with high temperatures .A transistor fabricated using nano tube at Holland can switched on and off with flow of a single electron. This reduces the size of the circuits in computers.Synthasis of carbon nano tubes



11)What is scanning electron microscope? Mention its three applications. (July2015)

A Practical device that relies on the wave characteristics of electrons, in which electron accelerated under a potential difference behaves like a wave.

Applications:

SEM is used to study.

(1) External morphology of biological organisms in the sub microscope.
(2) Chemical composition
(3) Crystalline structure. SEM is also used in forensic investigation.
(4) Used to study the corroded layers on metal surfaces
(5) Used to study biological specimens like pollen grains.

11).The distance between the two pressure sensors in a shock tube is 100mm.the time taken by shock wave to travel this distance is 200 microsecond. If the velocity of sound under the same conditions is 340m/s ,find the Mach number of the shock wave.

Solution:

Given, d=100mm, t=200 microsecond

V_{sound}=340 m/s

$$V_{shock} = \frac{d}{t} = \frac{100\times 10^{-3}}{200\times 10^{-6}} = 500 \text{m/s}$$

$$M = \frac{V_{shock}}{V_{sound}} = \frac{500}{340} = 1.47$$

Mach number of the shock wave is 1.47

12).Definemachnumber ,subsonic waves and supersonic waves. (July2015)

Mach number:

It is defined as the ratio of the speed of the object to the speed of the sound in the given medium, i.e.,

Mach * number $= \frac{objectspeed}{speedofsoundinthemedium}$

It is denoted as M. thus v is the object speed and the speed of sound in the medium is a, then, M $= \frac{v}{a}$.

Since it is a ratio of speeds, it doesn't have a unit as the name itself indicates, it is a pure number.

Subsonic waves:

If the speed of the mechanical wave or body moving in the fluid is lesser than that of sound then such a speed is referred to as subsonic and the wave is a subsonic wave. All subsonic waves have mach no<1.

The speeds of almost all the vehicles such as motor cars or trains that we see moving on the road fall in the subsonic category. The speeds of flight of birds is also subsonic.

For a body moving with subsonic speed, the sound emitted by it manages to move ahead &away from the body since it is faster than the body

Supersonicwaves:

Supersonic waves are mechanical waves which travels with speeds greater than that of sound, i.e. , with speeds for which, Mach number>1.

13). Discuss the basics of conservation of mass, momentum and energy.(July2015)

We know that conservation means the maintenance of certain quantities unchanged with physical process.

Law of conservation of mass: The total mass of any isolated system remains unchanged or constant and is independent of any chemical and physical changes that could occur within the system.

That is $\frac{dm}{dt} = \dot{m} = const$ in a physical system.

Law of conservation of momentum: In a closed or isolated system, the total moment remains constant. Or when two bodies collide with an isolated system, the total momentum of objects before collision equal to the after collision.

$\dot{P_1}$ □ $\frac{dmv_1}{dt}$ □ 0 ie,mv_1□ const

$\dot{P_2}$ □ $\frac{dmv_2}{dt}$ □ 0 ie, mv_2□ const

p_1 □ Momen~~tum~~ before collision

p_2 □ Momentum after collision

Law of conservation of energy: The total energy of the isolated system or closed system is constant and it is independent of any changes occurring within the system. Let total energy of the system per unit mass consist of the internal energy U, kinetic energy is K , potential energy is V and work done by the pressure P,

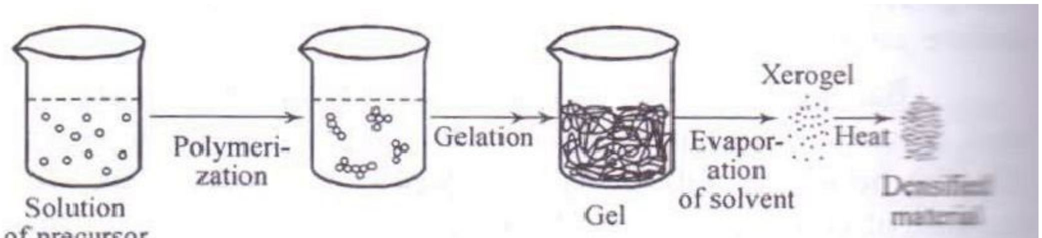
Then conservation of energy can be written as

$$\square_1 \square u_1^2 \square \square_2 \square u_2^2 \text{ - -}\square 3\square$$

14) Explain the Sol-Gel method of preparing nanomaterial's (July2015)

Sol-Gel technique: The sol is a name of a colloidal solution made of solid particles few hundred nm in diameter, suspended in a liquid phase and the gel can be considered as a solid macromolecule immersed in a solvent.

The sol-gel process is a wet-chemical based self-assembly process for nanomaterial formation. (Also known as Chemical Solution Deposition) widely used recently in the fields of materials science and ceramic engineering. Such methods are used primarily for the fabrication of materials (typically a metal oxide) starting from a chemical solution (sol, short for solution) which acts as the precursor for an integrated network (or gel) of either discrete particles or network polymers.



SOL-GEL PROCESS

Fig. 6

15). In scanning electron microscope, electrons are accelerated by an anode potential difference 60kilo volt. Estimate the wavelength of the electrons in the scanning beam. Given, V=60 × 10^3**(July2015)**

$$\lambda=\frac{\text{⍰}}{\sqrt{2meV}}=\frac{12.26\times 10^{-6}}{\sqrt{V}}=\frac{12.26\times 10^{-6}}{\sqrt{60\times 10^{3}}}=5.06\text{⍰}\ 10^{-12}\,\text{m}$$