Engineering Mathematics - IV VTU CBCS Question Paper Set 2018

CBCS Scheme

USN | | | | | | | | | | |

15MAT41

Fourth Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: 1. Answer any FIVE full questions, choosing one full question from each module.
2. Use of statistical tables is permitted.

Module-1

1 a. Employ Taylor's series method to find y at x = 0.1. Correct to four decimal places given $\frac{dy}{dx} = 2y + 3e^x$; y(0) = 0. **(05 Marks)**

b. Using Runge Kutta method of order 4, find y(0.2) for $\frac{dy}{dx} = \frac{y-x}{y+x}$; y(0) = 1, taking h = 0.2. **(05 Marks)**

c. If $y' = 2e^x - y$; y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040 and y(0.3) = 2.090. Find y(0.4) using Milne's predictor corrector formula. Apply corrector formula twice. **(06 Marks)**

OR

2 a. Use Taylor's series method to find y(4.1) given that $(x^2 + y)y' = 1$ and y(4) = 4. **(05 Marks)**

b. Using modified Euler's method find y at x = 0.1, given $y' = 3x + \frac{y}{2}$ with y(0) = 1, h = 0.1. Perform two iterations. **(05 Marks)**

c. Find y at x = 0.4 given $y' + y + xy^2 = 0$ and $y_0 = 1$, $y_1 = 0.9008$, $y_2 = 0.8066$, $y_3 = 0.722$ taking h = 0.1 using Adams-Bashforth method. Apply corrector formula twice. **(06 Marks)**

Module-2

3 a. Given $y'' = xy'^2 - y^2$ find y at x = 0.2 correct to four decimal places, given y = 1 and y' = 0 when x = 0, using R-K method. **(05 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$, then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(05 Marks)**

c. If $x^3 + 2x^2 - x + 1 = ap_0(x) + bp_1(x) + cp_2(x) + dp_3(x)$ then, find the values of a, b, c, d. **(06 Marks)**

OR

4 a. Apply Milne's method to compute y(0.8) given that $y'' = 1 - 2yy'$ and the table.

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

Apply corrector formula twice. **(05 Marks)**

b. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(05 Marks)**

c. Derive Rodrigue's formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2 - 1)^n\right]$. **(06 Marks)**

Module-3

5 a. Define analytic function and obtain Cauchy Riemann equation in Cartesian form. **(05 Marks)**

b. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$; c is the circle |z| = 3 by using theorem Cauchy's residue. **(05 Marks)**

c. Discuss the transformation $w = e^z$ with respect to straight line parallel to x and y axis. **(06 Marks)**

OR

6 a. Find the analytic function whose real part is $u = \frac{x^4 y^4 - 2x}{x^2 + y^2}$. **(05 Marks)**

b. State and prove Cauchy's integral formula. **(05 Marks)**

c. Find the bilinear transformation which maps the points z = 1, i, –1 into w = 2, i, –2. **(06 Marks)**

Module-4

7 a. Find the constant c, such that the function $f(x) = \begin{cases} cx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$ is a p.d.f. Also compute $p(1 < x < 2)$, $p(x \le 1)$, $p(x > 1)$. **(05 Marks)**

b. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals, more than two will get a bad reaction. **(05 Marks)**

c. x and y are independent random variables, x take the values 1, 2 with probability 0.7; 0.3 and y take the values -2, 5, 8 with probabilities 0.3, 0.5, 0.2. Find the joint distribution of x and y hence find cov(x, y). **(06 Marks)**

OR

8 a. Obtain mean and variance of binomial distribution. **(05 Marks)**

b. The length of telephone conservation in a booth has been an exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from this booth (i) ends less than 5 minutes. (ii) between 5 and 10 minutes. **(05 Marks)**

c. The joint distribution of two discrete variables x and y is f(x, y) = k(2x + y) where x and y are integers such that $0 \le x \le 2$; $0 \le y \le 3$. Find: (i) The value of k; (ii) Marginal distributions of x and y; (iii) Are x and y independent? **(06 Marks)**

Module-5

9 a. Explain the terms: (i) Null hypothesis; (ii) Type I and type II errors; (iii) Significance level. **(05 Marks)**

b. A die thrown 9000 times and a throw of 3 or 4 was observed 3240 times. Is it reasonable to think that the die is an unbiased one? **(05 Marks)**

c. Find the unique fixed probability vector for the regular Stochastic matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

(06 Marks)

OR

10 a. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure. ($t_{0.05}$ for 11 d.f = 2.201) **(05 Marks)**

b. It has been found that the mean breaking strength of a particular brand of thread is 275.6 gms with σ = 39.7 gms. A sample of 36 pieces of thread showed a mean breaking strength of 253.2 gms. Test the claim at 1+.. and 5–/. level of significance. **(05 Marks)**

c. A man's smoking habits are as follows. If he smokes filter cigarettes one week, he switches to non filter cigarettes the next week with probability 0.2. One the other hand, if he smokes non filter cigarettes one week there is a probability of 0.7 that he will smoke non filter cigarettes the next week as well. In the long run how often does he smoke filter cigarettes? **(06 Marks)**

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15MAT41

Fourth Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics–IV

Time: 3 hrs. Max. Marks: 80

Note: 1. ***Answer FIVE full questions, choosing one full question from each module.***
2. ***Use of statistical tables are permitted.***

Module-1

1 a. Find by Taylor's series method the value of y at x = 0.1 from $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ (upto 4th degree term). **(05 Marks)**

b. The following table gives the solution of $5xy' + y^2 - 2 = 0$. Find the value of y at x = 4.5 using Milne's predictor and corrector formulae. **(05 Marks)**

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187

c. Using Euler's modified method. Obtain a solution of the equation $\frac{dy}{dx} = x + \left|\sqrt{y}\right|$, with initial conditions y = 1 at x = 0, for the range $0 \le x \le 0.4$ in steps of 0.2. **(06 Marks)**

OR

2 a. Using modified Euler's method find y(20.2) and y(20.4) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with $y(20) = 5$ taking h = 0.2. **(05 Marks)**

b. Given $\frac{dy}{dx} = x^2(1+y)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) =1.548, y(1.3) = 1.979. Evaluate y(1.4) by Adams-Bashforth method. **(05 Marks)**

c. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at x = 0.2 by taking h = 0.2 **(06 Marks)**

Module-2

3 a. Obtain the solution of the equation $2\frac{d^2y}{dx^2} = ux + \frac{dy}{dx}$ by computing the value of the dependent variable corresponding to the value 1.4 of the independent variable by applying Milne's method using the following data: **(05 Marks)**

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y′	2	2.3178	2.6725	3.0657

b. Express $f(x) = 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. **(05 Marks)**

c. Obtain the series solution of Bessel's differential equation $x^2y'' + xy' + (x^2 + n^2)y = 0$ **(06 Marks)**

OR

4 a. By Runge-Kutta method solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2. Correct to four decimal places using the initial conditions y = 1 and $y' = 0$ at x = 0, h = 0.2. **(05 Marks)**

b. Prove that $J_{+\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ **(05 Marks)**

c. Prove the Rodrigues formula,

$$\rho_n(x) = \frac{1}{2^n n!} \frac{d^n (x^2 - 1)^n}{dx^n}$$ **(06 Marks)**

Module-3

5 a. State and prove Cauchy's-Riemann equation in polar form. **(05 Marks)**

b. Discuss the transformation $W = e^z$. **(05 Marks)**

c. Evaluate $\int_C \left\{ \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)^2 (z-2)} \right\} dz$

using Cauchy's residue theorem where 'C' is the circle $|z| = 3$ **(06 Marks)**

OR

6 a. Find the analytic function whose real part is, $\frac{\sin 2x}{\cosh 2y - \cos 2x}$. **(05 Marks)**

b. State and prove Cauchy's integral formula. **(05 Marks)**

c. Find the bilinear transformation which maps z = ∞, i, 0 into ω = -1, -i, 1. Also find the fixed points of the transformation. **(06 Marks)**

Module-4

7 a. Find the mean and standard deviation of Poisson distribution. **(05 Marks)**

b. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and S.D of 60 hours. Estimate the number of bulbs likely to burn for,

(i) more than 2150 hours.
(ii) less than 1950 hours
(iii) more than 1920 hours and less than 2160 hours.

[A(1.833) = 0.4664, A(1.5) = 0.4332, A(2) = 0.4772] **(05 Marks)**

c. The joint probability distribution of two random variables x and y is as follows:

x/y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Determine:

(i) Marginal distribution of x and y.
(ii) Covariance of x and y
(iii) Correlaiton of x and y. **(06 Marks)**

OR

8 a. The probability that a pen manufactured by a factory be defective is $\frac{1}{10}$. If 12 such pens are manufactured what is the probability that, (i) Exactly 2 are defective (ii) at least 2 are defective (iii) none of them are defective. **(05 Marks)**

b. Derive the expressions for mean and variance of binomial distribution. **(05 Marks)**

c. A random variable X take the values -3, -2, -1, 0, 1, 2, 3 such that $P(x = 0) = P(x < 0)$ and $P(x = -3) = P(x = -2) = P(x = -1) = P(x = 1) = P(x = 2) = P(x = 3)$. Find the probability distribution. **(06 Marks)**

Module-5

9 a. In 324 throws of a six faced 'die' an odd number turned up 181 times. Is it reasonable to think that the 'die' is an unbiased one? **(05 Marks)**

b. Two horses A and B were tested according to the time (in seconds) to run a particular race with the following results:

Horse A:	28	30	32	33	33	29	34
Horse B:	29	30	30	24	27	29	

Test whether you can discriminate between the two horses.($t_{0.05}$=2.2 and $t_{0.02}$=2.72 for 11 d.f) **(05 Marks)**

c. Find the unique fixed probability vector for the regular stochastic matrix, $A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$ **(06 Marks)**

OR

10 a. Define the terms: (i) Null hypothesis (ii) Type–I and Type–II error (iii) Confidence limits. **(05 Marks)**

b. Prove that the Markov chain whose t.p.m $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducible. Find the corresponding stationary probability vector. **(05 Marks)**

c. Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws (i) A has the ball. (ii) B has the ball. (iii) C has the ball. **(06 Marks)**

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